Info

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Homogeneous Riemannian manifolds

Definition. A Riemannian manifold (M, g) is a **homogeneous Riemannian manifold** if $\mathrm{Isom}(M, g)$ acts transitively on M.

These homogeneous Riemannian manifolds

- have constant *scalar* curvature

Homogeneous Riemannian manifolds are complete.

Thus we have

- $$\begin{aligned} {}^{G}\!/_{K} &\cong_G M \\ gK &\mapsto g(p) \end{aligned}$$

  whose derivative at i gives

  $$^{\mathfrak{g}}\!/_{\mathfrak{k}} \cong T_pM$$

- and we have the **exponential** map for geodesics

  $$\exp_p : T_pM \to M$$

Therefore, have a diagram

$$\begin{tikzcd}
\begin{matrix}\mathfrak{g}\\ X\end{matrix} \arrow[r, "\exp"] \arrow[d] & \begin{matrix}G\\ \exp(X)\end{matrix} \arrow[dr, bend left, "\mathrm{ev}_p"] & \\
\begin{matrix}\mathfrak{g}/\mathfrak{k}\\ X\end{matrix} \arrow[r, "\cong"] & \begin{matrix}T_pM\\ X\end{matrix} & \begin{matrix}M\\ \exp(X)(p)\end{matrix}
\end{tikzcd}$$

$$\begin{tikzcd}
\begin{matrix}T_pM\\ X\end{matrix} \arrow[r, "\exp_p"] & \begin{matrix}M\\ \exp_p(X)\end{matrix}
\end{tikzcd}$$

where the two exponentials may **not** be the same!

Current note has 0 direct children and 0 total descendants.

And it has 14 siblings.