Info

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Fundamental group of the complement of the knot

Definition. Fundamental group of the complement of the knot is

$$\pi_1(S^3 \setminus K)$$

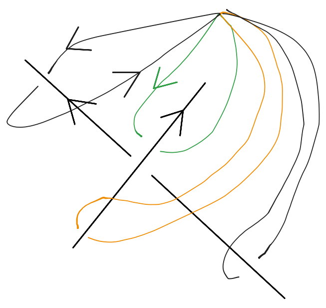
$$\pi_1(\mathbb{R}^3 \setminus K) \cong \pi_1(S^3 \setminus K)$$

We write

$$S^3 \setminus K = B \cup \mathbb{R}^3 \setminus K$$

as the union of $\mathbb{R}^3 \setminus K$ and an open ball B formed by the compactification point together with the complement of a large closed ball in $\mathbb{R}^3$ containing K. Both B and $B \cap (\mathbb{R}^3 \setminus K)$ are simply-connected, the latter space being homeomorphic to $S^2 \times \mathbb{R}$. Hence van Kampen's theorem implies that the inclusion $\mathbb{R}^3 \setminus K \to S^3 \setminus K$ induces an isomorphism on π_1.

For each arc in a knot diagram, we shall get one generator, say a_i



$$a_{i_1}^{-1} a_{i_2} a_{i_1} = a_{i_3}$$

The abelianization of the fundamental group of the complement of a knot is isomorphic to $\mathbb{Z}$.

Current note has 0 direct children and 0 total descendants.

- stem
  - Knots
    - Knots in $\mathbb{R}^3$
      - Fundamental group of the complement of the knot

And it has 5 siblings.

- stem
  - Knots
    - Knots in $\mathbb{R}^3$
      - Left trefoil knot
      - Complement of knots in $\mathbb{R}^3$ or S^3
      - Genus of a knot
      - Fundamental group of the complement of the knot
      - Linking number invariant of links