Info

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Manifold of probability measures

Also called a statistical manifold.

Definition. Manifold of probability measures

Let X be a measure space measure space then M is a **manifold of probability measures on** M if M is a smooth manifold with a smooth? map

$$\pi : M \to \mathbb{P}(X)$$

Definition.

$$\begin{aligned} \langle\ \rangle : L^1(X, \mathcal{A}_X, \mu) &\to \mathcal{C}^\infty(M) \\ A \mapsto \langle A \rangle_{\pi_{(-)}} &:= \int_{x \in X} A(x)\pi_{(-)}(x) \end{aligned}$$

Definition.

$$\begin{aligned} \mathrm{var} : L^1(X, \mathcal{A}_X, \mu) &\to \mathcal{C}^\infty(M) \\ \mathrm{var}_{(-)}(A) &:= \langle (A - \langle A \rangle_{(-)})^2 \rangle_{(-)} \end{aligned}$$

Definition. Relative information entropy metric tensor

Using

> **Definition.** Relative information entropy
>
> $$\begin{aligned} I : \mathscr{P}(X) \times \mathscr{P}(X) &\to \mathbb{R} \\ I(p,q) &:= \int_X p(x) \log\left(\frac{p(x)}{q(x)}\right) \end{aligned}$$

we have the metric tensor on M

$$g_p(\partial_{x_i}, \partial_{x_j}) := \int_X \partial_{x_i} \log \pi_p \partial_{x_j} \log \pi_p$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Measure spaces
    - b
      - Manifold of probability measures

And it has 3 siblings.

- structures based on sets
  - Measure spaces
    - b
      - Manifold of Gibbs distribution
      - Manifold of probability measures
      - Space of probability measures