Info

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Space of continuous and continuously differentiable functions on $\mathbb{R}$

Definition. Space of continuously differentiable functions $\mathcal{C}^k(I,\mathbb{R})$ on intervals $I \subseteq \mathbb{R}$

Let $I \subseteq \mathbb{R}$ be an interval. Then the $\mathbb{R}$-algebra of **continuously differentiable functions** is

$$\mathcal{C}^k(I,\mathbb{R}) := \{f : I \to \mathbb{R} : f^{(k)} : I \to \mathbb{R} \text{ exists and is continuous}\}$$

Definition. Derivative of functions on $\mathbb{R}$

Let $f : I(\text{interval}) \subseteq \mathbb{R} \to \mathbb{R}$ be a function and $x \in I$. Then

$$\begin{aligned} I \setminus \{x\} &\to \mathbb{R} \\ t &\mapsto \frac{f(t)-f(x)}{t-x} \end{aligned}$$

is well-defined. Then $f'(x) \in \mathbb{R} \cup \{\infty, -\infty\}$ is defined to be

$$f'(x) := \lim_{(a,b)\setminus\{x\} \ni t \to x} \frac{f(t)-f(x)}{t-x}$$

whenever the right side exists in $\mathbb{R} \cup \{\infty, -\infty\}$.
The function f is called **differentiable** at p if $f'(p) \in \mathbb{R}$.
Higher derivatives are defined iteratively

$$\begin{aligned} f^{(0)} &:= f \\ f^{(n)}(x) &:= (f^{(n-1)})'(x) \end{aligned}$$

whenever they exist.

- space.R.1.f R.diffable.space.open 1 $\mathcal{C}^1((a,b),\mathbb{R})$
- $\mathcal{C}^1([a,b],\mathbb{R})$
- $\mathcal{C}^k_c((a,b),\mathbb{R})$

Current note has 2 direct children and 4 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Space of continuous and continuously differentiable functions on $\mathbb{R}$
        - $\mathcal{C}^1([a,b],\mathbb{R})$
          - Derivative operator on $\mathcal{C}^1[a,b]$
          - $(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty)$
        - $\mathcal{C}^k_c((a,b),\mathbb{R})$

And it has 10 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Functions $(a,b) \to \mathbb{R}$ differentiable at a point
      - Comparing a function and it derivative
      - Absolutely continuous functions on $[a,b] \leftrightarrow \int_{[a,-]}(L^1[a,b])$
      - Distributional derivatives
      - Double derivative/Laplace operator on the circle
      - Fractional derivative
      - Infinite limit of derivatives
      - Space of continuous and continuously differentiable functions on $\mathbb{R}$
      - Derivative of maps $\mathbb{R}^n \to \mathbb{R}^m$
      - Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$