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Sheaf on open sets of a topological space

Definition. Sheaf on open sets of a topological space

A *functor* from the opposite of the category of open sets on X

$$\mathcal{S} : (\text{Open}(X), \subseteq)^{\text{op}} \rightarrow \mathbf{C}$$

is a **pre-sheaf** of $\mathbf{C}$ on X , where for every pair of open sets $U \subseteq V$, there is a morphism

$$\text{res}_{V,U} : \mathcal{S}(V) \rightarrow \mathcal{S}(U)$$

in $\mathbf{C}$. A presheaf is a **sheaf** if for every open cover $\{U_i\}_{i \in I}$ of $U \subseteq X$ we have

- (*identity*)

$$f \in \mathcal{S}(U), \forall i \in I, \text{res}_{U,U_i}(f) = 0 \implies f = 0$$

- (*gluability*) Given $f_i \in \mathcal{S}(U_i)$ for all $i \in I$, such that

$$\text{res}_{U_i,U_i \cap U_j}(f_i) = \text{res}_{U_j,U_i \cap U_j}(f_j)$$

then

$$\begin{aligned} \exists f \in \mathcal{S}(U) \\ \text{res}_{U,U_i}(f) = f_i \end{aligned}$$

Roughly speaking, sheaves are those functors, where local data can be glued to global data.

- morphism

- “2305 Smooth is Top.N-6#^a64538” could not be found.

stalk

$$\mathcal{F}_x := \frac{\{\sigma \in \mathcal{F}(U) : U \subseteq X\}}{\sim}$$

- cohomologies

- (Grotheindiek definition of sheaf cohomology)
- the complex $C^\bullet(\mathfrak{U}, \mathcal{F}), \delta$ with coboundary for a given cover $\mathfrak{U}$

$$H^k(C^\bullet(\mathfrak{U}, \mathcal{F}), \delta)$$

- direct limit of the previous (Cech cohomology)

$$\lim_{\mathfrak{U}} H^k(C^\bullet(\mathfrak{U}, \mathcal{F}), \delta)$$

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