Info

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Sheaf on open sets of a topological space

Definition. Sheaf on open sets of a topological space

A *functor* from the opposite of the category of open sets on X

$$\mathscr{S} : (\mathrm{Open}(X), \subseteq)^{\mathsf{op}} \to \mathsf{C}$$

is a **pre-sheaf** of C on X, where for every pair of open sets $U \subseteq V$, there is a morphism

$$\mathrm{res}_{V,U} : \mathscr{S}(V) \to \mathscr{S}(U)$$

in C. A presheaf is a **sheaf** if for every open cover $\{U_i\}_{i\in I}$ of $U \subseteq X$ we have

- *(identity)*

$$f \in \mathscr{S}(U), \forall i \in I,\ \mathrm{res}_{U,U_i}(f) = 0 \implies f = 0$$

- *(gluability)* Given $f_i \in \mathscr{S}(U_i)$ for all $i \in I$, such that

$$\mathrm{res}_{U_i,U_i\cap U_j}(f_i) = \mathrm{res}_{U_j,U_i\cap U_j}(f_j)$$

  then

$$\begin{gathered}\exists f \in \mathscr{S}(U)\\ \mathrm{res}_{U,U_i}(f) = f_i\end{gathered}$$

Roughly speaking, sheaves are those functors, where local data can be glued to global data.

- morphism
  - "2305 Smooth is Top.N-6#^a64538" could not be found.

- [x] ~~stalk~~

$$\mathscr{F}_x := \frac{\{\sigma \in \mathscr{F}(U) : U \subseteq X\}}{\sim}$$

- cohomologies
  - (Grotheindiek definition of sheaf cohomology)
  - the complex $C^\bullet(\mathfrak{U}, \mathscr{F}), \delta$ with coboundary for a given cover $\mathfrak{U}$

$$H^k(C^\bullet(\mathfrak{U}, \mathscr{F}), \delta)$$

  - direct limit of the previous (Cech cohomology)

$$\lim_{\mathfrak{U}} H^k(C^\bullet(\mathfrak{U}, \mathscr{F}), \delta)$$

- Product of topological spaces
- Ringed topological spaces
- Sheaf on open sets of a topological space
- Closed subsets of a topological space
- Discrete subsets of topological spaces
- Types of topological spaces
- Locally compact Hausdorff spaces
- Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$