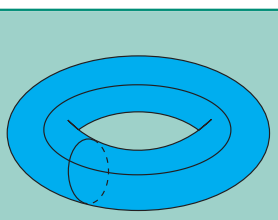
Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on November 5, 2023 3:41:21 PM,
was last modified on July 21, 2024 5:28:00 PM, and
was exported on September 7, 2026 3:04:04 PM.

Translation flows on the 2-torus

Definition.



Translation action on the 2-torus by itself

$$(T^2, +) \curvearrowright T^2$$
$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} \overset{\begin{bmatrix}v_1\\ v_2\end{bmatrix}}{\longmapsto} \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} + \begin{bmatrix}v_1\\ v_2\end{bmatrix}$$

$$\mathfrak{Lie}(T^2) \cong (\mathbb{R}^2, 0) \to \mathrm{Vec}(T^2)$$
$$e_1, e_2 \mapsto \frac{\partial}{\partial\theta_1}, \frac{\partial}{\partial\theta_2}$$

[1]

Flow of a general vector field

$$\omega_1 \frac{\partial}{\partial\theta_1} + \omega_2 \frac{\partial}{\partial\theta_2}$$

is given by

$$\exp\left(t\left(\omega_1 \frac{\partial}{\partial\theta_1} + \omega_2 \frac{\partial}{\partial\theta_2}\right)\right)\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} + t\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}$$

if	flow is	which gives a	that is coming from
$\omega_1/\omega_2 \in \mathbb{Q}$	periodic	S^1-action on T^2	composition $S^1 \hookrightarrow T^2 \curvearrowright T^2$
$\omega_1/\omega_2 \in \mathbb{R} \setminus \mathbb{Q}$	not periodic, minimal (orbits are dense)	$\mathbb{R}$-action on T^2	composition $\mathbb{R} \hookrightarrow T^2 \curvearrowright T^2$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - T^2
    - Translation flows on the 2-torus

And it has 2 siblings.

- stretchy
  - T^2
    - Torus identified with a glued rectangle
    - Translation flows on the 2-torus

---

1. 421Week6Lecture2.pdf (jhu.edu) ↩