Info

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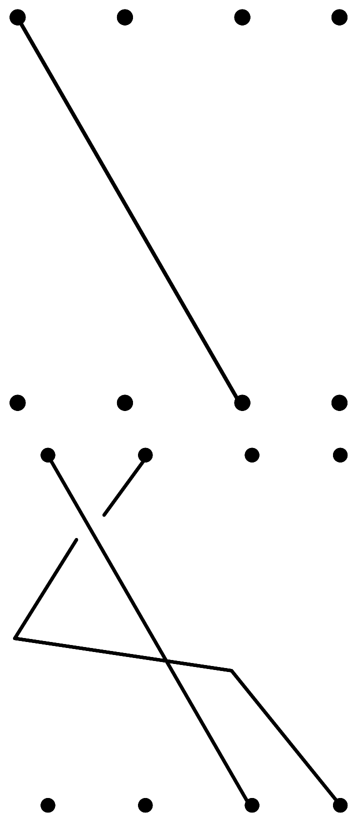
Braids

Let

$$A_i := (i, 0, 0)$$
$$B_i := (i, 0, -1)$$

A n-**braid** is a union of n polygonal segments each joining A_i to a B_j for $i, j \in \{1, \ldots, n\}$ such that

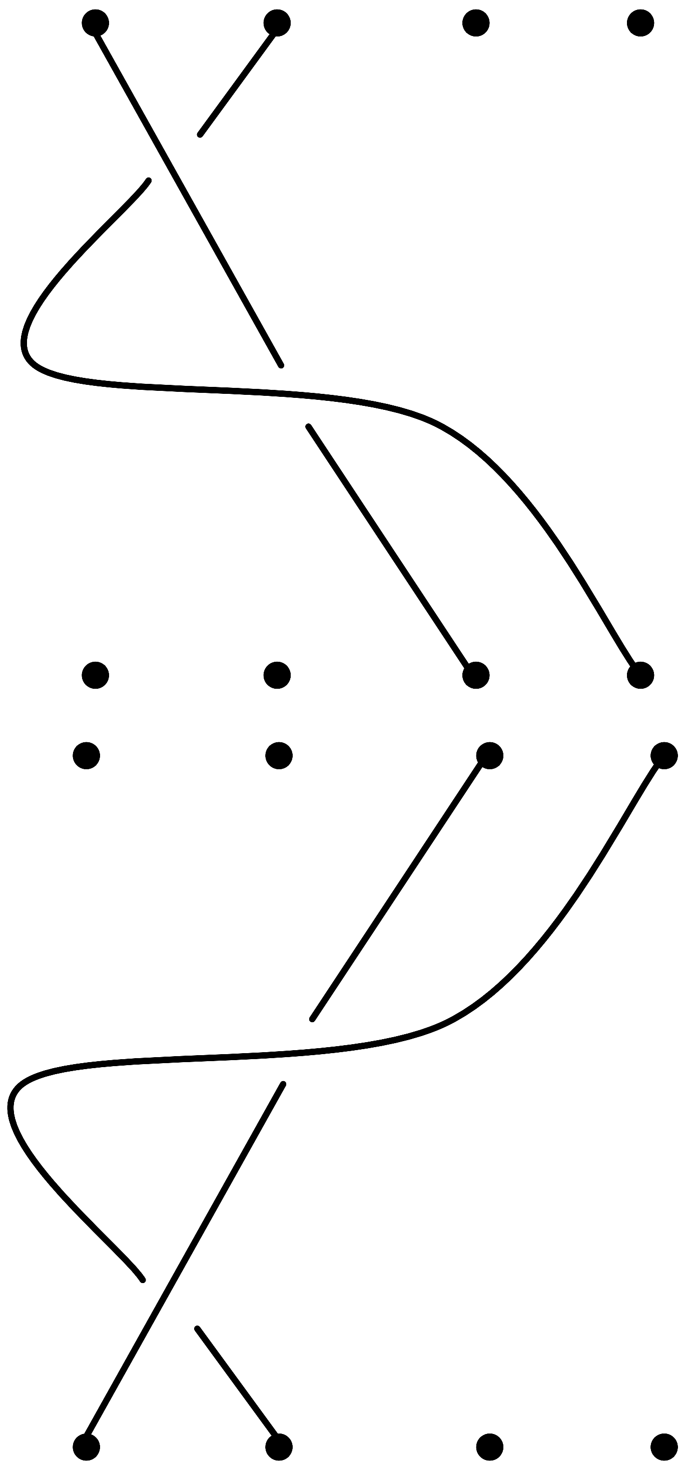
- they are pairwise disjoint
- each line segment intersects each $z = a$ plane only once for all $a \in [0, 1]$



Two such braids are **equivalent** if we can go from one to the other by *allowed* Δ-moves or ambient isotopy.

Let B_n denote the equivalence classes of n-braids.

B_n forms a group under "concatenation".



closure-isotopy and conjugaction

$$B_1 \cong \{i\}$$

$$B_2 \cong \langle e_1 \rangle \cong \mathbb{Z}$$

$$B_3 \cong \langle e_1, e_2 | e_1 e_2 e_1 = e_2 e_1 e_2 \rangle \cong \langle x, y | x^2 = y^3 \rangle$$
$$e_1 e_2 e_1 \longleftarrow x$$

$$\iota_n : B_n \to B_{n+1}$$

Let $b \in B_n$ and $b' \in B_{n'}$. Then the closures of b and b' are isotropic as links in $\mathbb{R}^3$

$$\iff$$

b and b' are related by a sequence of moves of the following types (I)

$$n = n',\ \exists x \in B_n : b = x^{-1} b' x$$

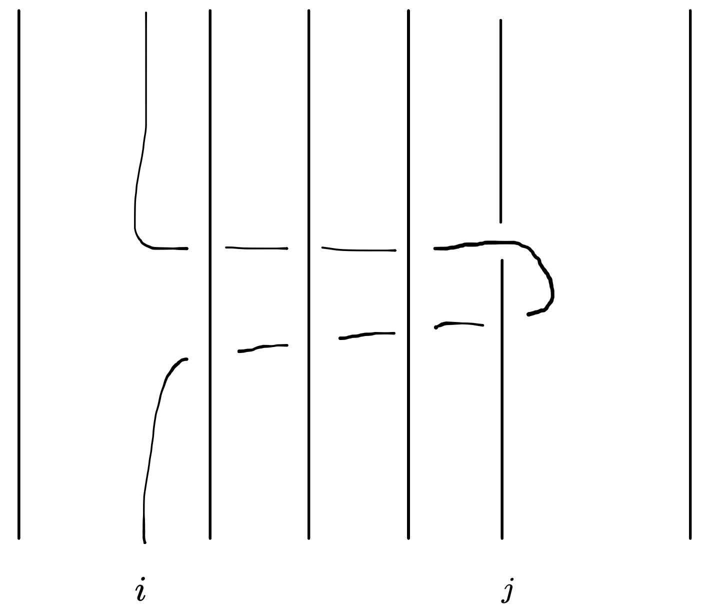
and (II)

$$b \in B_n, b' \in B_{n+1} \implies b' = b e_n$$

homomorphism to permutation group

$$\Theta_n : B_n \to S_n$$

where $P_n := \ker \Theta_n$ is called the **pure braid group**.



f_{ij}

$P_n \leq B_n$ is finitely generated by the elements

$$\{f_{ij} | 1 \leq i < j \leq n\}$$

(Proof is by induction)

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[1]

[2]

Current note has 0 direct children and 0 total descendants.

- stem
  - Knots
    - Braids

And it has 3 siblings.

- stem
  - Knots
    - Braids
    - Knot and link diagrams on $\mathbb{R}^2$
    - Knots in $\mathbb{R}^3$

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1. Prasolov & Sassinsky ↩
2. Braid groups - Kassel & Trauev ↩