Info

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Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset

Let $Q := \{q_n | n \in \mathbb{N}\} \subseteq \mathbb{R}^2$ be a countable, closed and discrete subset. Then there exists a homeomorphism

$$h : \mathbb{R}^2 \to \mathbb{R}^2$$

such that $f(q_n) = (n, 0)$ for all $n \in \mathbb{N}$.

Pick $a \in \mathbb{R}^2$ such that

$$a \in \mathbb{R}^2 \setminus \bigcup_{n,m\in\mathbb{N}} \overleftrightarrow{q_n q_m}$$

Then for each n the line segment $\overline{aq_n}$ is a compact set disjoint from $Q \setminus \{q_n\}$.

- For $n \in \mathbb{N}$ let

$$c(n, r) := \text{convex}(\overline{B_r(a)} \cup \overline{B_r(q_n)})$$

- For

$$c(n, r) \cap Q \subseteq \{q_1, \ldots, q_{m-1}\} \cup \{q_n\}$$

[1]

Current note has 0 direct children and 0 total descendants.

- stretchy
  - Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset

And it has 62 siblings.

- stretchy
  - Trefoil 3_1 knot
  - The only compact connected Lie group that can act faithfully on a torus is itself
  - Hecke algebras
  - $\overline{B^n}$
  - $\mathrm{BS}(1, 2)$
  - $(\mathbb{C}, +, \times)$
  - $D^* := D \setminus \{0\}$
  - Cantor set
  - Configuration space of N rigid points in $\mathbb{R}^3$
  - Free groups
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
  - H_2^3
  - $[a, b]$
  - Figure 8 knot
  - Lamplighter group on $\mathbb{Z}_2$
  - $\beta\mathbb{N}$
  - $\#_2\mathbb{R}P^2$
  - $\prod_{i\in\mathbb{N}} \{1, \ldots, n_i\}$
  - $\mathbb{R}P^n$
  - $(\mathbb{Q}, +, \times, <)$
  - $\mathbb{Q}_{\hat{p}}$
  - $\overline{\mathbb{Q}}$
  - $\mathbb{Q}_{\text{constr}}$
  - $\mathbb{Q}(\xi_n)$
  - $\mathcal{O}_{\overline{\mathbb{Q}}}$
  - $(\mathbb{R}, +, \times, <, \sup)$
  - Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset
  - $\mathbb{R}^2 \times S^1$
  - $\mathbb{R}^n$
  - $\mathbb{R} \times S^1$
  - $\mathbb{R} \times S^2$
  - S^1
  - $S^1 \wedge S^1$
  - $S^1 \times S^n$
  - S^2 and $\mathbb{C}\boldsymbol{P}^1$
  - Mapping torus of the antipodal map of S^2
  - S^3
  - S^n
  - I-bundle
  - Symmetric group
  - $\mathbb{T}$
  - T^2
  - T_g^2
  - $T_{0,0,3}^2$

- $\mathbb{R}^2 \setminus \{0\}$
- Three $T^2_{1,0,1}$ glued at their boundaries
- $T^2_{1,1,0}$
- T^2_2
- $T^2_g \times [0,1]$
- $\#_{\mathbb{N}} T^2$
- Thompson's group
- T^n
- 3-valent tree
- 4-valent tree
- $[0,1]^{\mathbb{N}}$ with product topology
- Whitehead manifolds
- $\mathbb{Z}$
- $\mathbb{Z}[\sqrt{m}]$
- $\mathbb{Z}[X]$
- $\mathbb{Z}^{\mathbb{N}}$
- $\mathbb{Z}[i]$
- $\mathbb{Z}[i\sqrt{5}]$

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1. https://math.stackexchange.com/a/1320582/1290493 ↩