Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on November 23, 2025 4:04:23 PM,
was last modified on November 23, 2025 4:26:27 PM, and
was exported on September 7, 2026 3:25:06 PM.

Isometries of CAT(0) spaces

6.7 Proposition. *Let X be a complete* CAT(0) *space, and let γ be an isometry of X. Then, γ is elliptic if and only if γ has a bounded orbit. And if γ^n is elliptic for some integer $n \neq 0$, then γ is elliptic.*

6.8 Theorem. *Let X be a* CAT(0) *space.*

(1) *An isometry γ of X is hyperbolic if and only if there exists a geodesic line $c : \mathbb{R} \to X$ which is translated non-trivially by γ, namely $\gamma.c(t) = c(t + a)$, for some $a > 0$. The set $c(\mathbb{R})$ is called an axis of γ. For any such axis, the number a is actually equal to $|\gamma|$.*

(2) *If X is complete and γ^m is hyperbolic for some integer $m \neq 0$, then γ is hyperbolic.*

Let γ be a hyperbolic isometry of X.

(3) *The axes of γ are parallel to each other and their union is* $\mathrm{Min}(\gamma)$.

(4) $\mathrm{Min}(\gamma)$ *is isometric to a product $Y \times \mathbb{R}$, and the restriction of γ to* $\mathrm{Min}(\gamma)$ *is of the form $(y, t) \mapsto (y, t + |\gamma|)$, where $y \in Y$, $t \in \mathbb{R}$.*

(5) *Every isometry α that commutes with γ leaves $\mathrm{Min}(\gamma) = Y \times \mathbb{R}$ invariant, and its restriction to $Y \times \mathbb{R}$ is of the form (α', α''), where α' is an isometry of Y and α'' a translation of $\mathbb{R}$.*
