Info

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Lagrangian flows

Definition. Critical points of Lagrangian actions

Let

$$L : \mathbb{R} \times TM \to \mathbb{R}$$

be a *time-dependent* smooth function on TM for a smooth manifold M. Then a curve

$$\gamma : [a, b] \to M$$

is a critical point of the functional

$$\mathcal{A}_L[\gamma] := \int_I L(t, \dot{\gamma}) \mathrm{d}t$$

if for every variation with fixed endpoints

$$\begin{aligned} \tilde{x} : (-\epsilon, \epsilon) \times [a, b] &\to M \\ \tilde{x}(0, -) &= x \\ \tilde{x}(-, a) &= x(a) \\ \tilde{x}(-, b) &= x(b) \end{aligned}$$

we have

$$(\mathrm{d}\mathcal{A}_L)(\tilde{x}) := \frac{\mathrm{d}}{\mathrm{d}s}\bigg|_{s=0} \mathcal{A}_L(\tilde{x}(s, -)) = 0$$

Let $L \in \mathcal{C}^\infty(\mathbb{R} \times TM)$, then a smooth curve

$$\gamma : [a, b] \to M$$

is a critical point of the Lagrangian action for L $\iff$ for any induced coordinates (q, v) on TM from any coordinates (q) on M, the *Euler-Lagrange equations* hold

$$\left(\frac{\partial L}{\partial q^k} - \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial v^k} \right)(t, \dot{\gamma}) = 0$$

for all $k \in \{1, \ldots, n\}$.

[1]

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Lagrangian flows

And it has 19 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Smooth Lie group action on a smooth manifold
      - Borel measures on a smooth manifold
      - G-bundles over smooth manifolds $G \curvearrowright P \to X$
      - Vector bundle on smooth manifolds
      - Curves in smooth manifolds
      - Density on smooth manifolds
      - Functions between smooth manifolds
      - Functions from smooth manifolds to $\mathbb{R}$
      - Flows on a smooth manifold
      - Locally $G \curvearrowright X$-manifolds
      - Smooth singular homology of a smooth manifold
      - Isomorphisms and automorphisms of smooth manifolds
      - Lagrangian flows
      - Local transformation rules
      - Moduli space of Riemannian metrics on a smooth manifold
      - Moduli space of quotients
      - Quotient manifolds
      - Submanifolds of smooth manifolds
      - Tangents on a smooth manifold

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1. asc.ohio-state.edu/terekcouto.1/texts/var_noether.pdf#page=1.11 ↩