

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on November 6, 2023 4:38:46 PM, was last modified on October 1, 2025 1:54:16 PM, and was exported on September 7, 2026 3:31:16 PM.

Lie algebra of a lie group

Definition. Lie algebra of a lie group

Lie algebra $\text{Lie}(G)$ **of a lie group** G is defined as the $\mathbb{R}$ -vector space $(T_iG, +, \cdot)$ with the lie bracket defined using the isomorphism

Let G be a Lie group. The evaluation map

$$\begin{aligned} \text{eval}_i : \text{LVec}(M) &\rightarrow T_iG \\ X &\mapsto X_i \end{aligned}$$

is a $\mathbb{R}$ -linear isomorphism.

Definition. Lie algebra of a lie group functor

$$\begin{aligned} \text{Lie} : \text{Man}_{\mathbb{R}}\text{Grp} &\rightarrow \text{Alg}_{\mathbb{R}}\text{Lie} \\ G &\mapsto \text{Lie}(G) \\ f : G_1 \rightarrow G_2 &\mapsto \text{d}_i f : \text{Lie}(G_1) \rightarrow \text{Lie}(G_2) \end{aligned}$$

the correspondence

- For every Lie $\mathbb{R}$ -algebra, there **exists** a simply connected Lie group.
- Two simply connected Lie groups have isomorphic Lie algebras



they are isomorphic as Lie groups.

- For any Lie group, it's universal cover is a simply connected (naturally) Lie group ^[1]

[2]

Current note has 2 direct children and 2 total descendants.

- [structures based on sets](#)
 - [manifolds](#)
 - [ℝ-manifold group \(Lie groups\)](#)
 - [Lie algebra of a lie group](#)
 - [Exponential map from lie algebra of a lie group](#)
 - [Left-invariant vector fields](#)

And it has 18 siblings.

- [structures based on sets](#)
 - [manifolds](#)
 - [ℝ-manifold group \(Lie groups\)](#)
 - [Adjoint representation of a Lie group](#)
 - [Conjugation action of a Lie group on itself](#)
 - [Classification of Lie groups](#)
 - [Compact Lie groups](#)
 - [Connected Lie groups with discrete center](#)
 - [Connected Lie groups with non-surjective exponential map](#)
 - [Discrete subgroups of Lie groups](#)
 - [Smooth maps onto a Lie group](#)
 - [Lie groups with surjective exponential map](#)
 - [Maurer-Cartan form](#)
 - [Left-invariant Lagrangian on a Lie group](#)
 - [Lie algebra of a lie group](#)
 - [Bi-invariant metrics on Lie groups](#)
 - [Left invariant Riemannian metric on a Lie group](#)
 - [Semi-simple ℝ-Lie groups](#)
 - [Almost simple groups](#)
 - [Virtual subgroups of Lie groups](#)
 - [Lie group with dilations](#)

1. [textbook recommendation - Classification of \(compact\) Lie groups - MathOverflow](#) ↔
2. <https://www.math.ucla.edu/~vsv/liegroups2007/9.pdf> ↔