ⓘ Info

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Hamiltonian flows on the sphere S^2

The Hamiltonian vector fields for the area symplectic form

> **Definition.** The area form on round S^2
>
> The area form λ_{S^2} on the 2-sphere is the *volume form* induced by the *spherical metric* $S^2 \hookrightarrow \mathbb{R}^3$.

which

> - In cylindrical polar coordinates,
>
> $$\lambda_{S^2} = \mathrm{d}\theta \wedge \mathrm{d}z$$

so

$$\begin{aligned} &\lambda_{\mathsf{S}}(X, -) = -\mathrm{d}H(-) \\ \implies & -\frac{\partial f}{\partial z}\frac{\partial}{\partial \theta} + \frac{\partial f}{\partial \theta}\frac{\partial}{\partial z} \end{aligned}$$

Thus,

$$-(\omega^{-1}\mathrm{d}f)_p = J_p \operatorname{grad}_p(f) = p \times \operatorname{grad}_p(f)$$

- Area preserving flow $\iff$ the generating vector field is Hamiltonian $\iff$ $\mathrm{d}(\iota_X\omega) = 0$
- examples
  - The Hamiltonian vector field of $(\theta, z) \mapsto z$ is $-\partial_\theta$

- A vector field is Hamiltonian $\iff$ symplectic as S^2 is simply connected.

Current note has 0 direct children and 0 total descendants.

- stretchy
  - S^2 and $\mathbb{C}\boldsymbol{P}^1$
    - Hamiltonian flows on the sphere S^2

And it has 10 siblings.

- stretchy
  - S^2 and $\mathbb{C}\boldsymbol{P}^1$
    - Almost complex structure on S^2 induced from spherical metric and its area form
    - Borsuk-Ulam theorem
    - Cylindrical coordinates on S^2
    - Holomorphic vector bundles on $\mathbb{C}\boldsymbol{P}^1$
    - Round metric on S^2
    - The area form on round S^2
    - Stereographic coordinates on the sphere S^2
    - TS^2
    - Vector fields on S^2
    - Hamiltonian flows on the sphere S^2