

Info

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on March 2, 2026 3:41:34 PM, was last modified on September 7, 2026 1:11:08 PM, and was exported on September 7, 2026 3:09:57 PM.

$$\frown : L^2(0, \infty) \cong_{\text{Hilb}} \mathcal{O}^2(H^2_{\mathbb{U}})$$

Definition.

$$\mathcal{O}^p(H^2_{\mathbb{U}}) := \left\{ f \in \mathcal{O} \left| \sup_{y \in (0, \infty)} \int_{\mathbb{R} + iy} |f|^p < \infty \right. \right\}$$

[1]

Let $f \in L^2(0, \infty) \leq L^2(\mathbb{R})$ and $z \in H^2_{\mathbb{U}}$ consider

$$\begin{aligned} |f(t) \exp(itz)| &= |f(t) e^{-t\Im(z)}| \\ \implies t \mapsto f(t) \exp(itz) &\in L^2(0, \infty) \end{aligned}$$

so we have the Fourier transform

$$\hat{f}(z) := \int_{(0, \infty)} f(t) \exp(itz)$$

for all $z \in H^2_{\mathbb{U}}$.

(Paley-Wiener)

$$\frown : L^2(0, \infty) \cong_{\text{Hilb}} \mathcal{O}^2(H^2_{\mathbb{U}})$$

[2]

For any $\delta > 0$ and $y \geq \delta$ we have

$\left| \int_{x \in (0, \infty)} f(x) e^{2\pi i x z} \right| \leq \int_{x \in (0, \infty)} |f(x)| e^{2\pi x |z|}$

$\leq \sqrt{\int_{(0, \infty)} |f|^2} \sqrt{\int_{x \in (0, \infty)} e^{-4\pi x \delta}}$

$\begin{array}{ccc} ? & \longrightarrow & \mathcal{O}^2(H^2_{\mathbb{U}}) \\ \downarrow & \nearrow \frown & \downarrow \Im(z) \rightarrow 0 \\ L^2(0, \infty) & \xhookrightarrow{\frown} & L^2(\mathbb{R}) \end{array}$

$\mathcal{O}^2(H^2_{\mathbb{U}})$

$\downarrow \Im(z) \rightarrow 0$

$L^2(0, \infty) \xrightarrow{\frown} L^2(\mathbb{R})$

Current note has 0 direct children and 0 total descendants.

stem

subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$

Fourier

$\frown : L^2(0, \infty) \cong_{\text{Hilb}} \mathcal{O}^2(H^2_{\mathbb{U}})$

And it has 10 siblings.

stem

subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$

Fourier

Fourier transform on $L^2(-A, A) \leq L^2(\mathbb{R})$

$\frown : L^2(0, \infty) \cong_{\text{Hilb}} \mathcal{O}^2(H^2_{\mathbb{U}})$

Fourier transform on $\mathbb{R}^n$

Fourier transform on S^1 , Fourier series on $[0, 1]$

Functions on S^1 with absolutely converging Fourier series, $\check{L}^1(S^1)$

Fourier transform of distributions on S^1

$\frown : L^1[0, 1] \rightarrow \mathcal{C}_0(\mathbb{Z}, \mathbb{C})$

$\frown : L^2[0, 1] \cong_{\text{Hilb}} l^2(\mathbb{Z}, \mathbb{C})$

Fourier transform of measurable subsets

Unsharpness principles

1. [book.pdf](#)↔

2. Definition.

$$\mathcal{O}^p(H^2_{\mathbb{U}}) := \left\{ f \in \mathcal{O} \left| \sup_{y \in (0, \infty)} \int_{\mathbb{R} + iy} |f|^p < \infty \right. \right\}$$

