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Differentiable functions

We have the *classical* way to differentiate functions.

Definition. Derivative of functions on $\mathbb{R}$

Let $f : I(\text{interval}) \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a function and $x \in I$. Then

$$I \setminus \{x\} \rightarrow \mathbb{R}$$
$$t \mapsto \frac{f(t) - f(x)}{t - x}$$

is well-defined. Then $f'(x) \in \mathbb{R} \cup \{\infty, -\infty\}$ is defined to be

$$f'(x) := \lim_{(a,b) \setminus \{x\} \ni t \rightarrow x} \frac{f(t) - f(x)}{t - x}$$

whenever the right side exists in $\mathbb{R} \cup \{\infty, -\infty\}$.
The function f is called **differentiable** at p if $f'(p) \in \mathbb{R}$.
Higher derivatives are defined iteratively

$$f^{(0)} := f$$
$$f^{(n)}(x) := (f^{(n-1)})'(x)$$

whenever they exist.

This may generalize to: f is *weakly differentiable* on $[a, b]$ if there is a $\psi \in L^1[a, b]$ such that

$$f(x) = c_1 + \int_{[a,x]} \psi$$

This is equivalent to f being [absolutely continuous](#).

This generalizes further to

Definition. Distributional derivatives of L^1_{loc} functions

Let $U(\text{open}) \subseteq \mathbb{R}^n$ be an open set and $f \in L^1_{\text{loc}}(U)$. Then $w \in L^1_{\text{loc}}$ is a **distributional derivative** of f in $\hat{x}_j$ direction

$$\int \partial_j f = w \text{ if } \forall \varphi \in \mathcal{C}^\infty_c(U), \int_U w \varphi = - \int_U f \partial_j \varphi$$

Current note has 10 direct children and 18 total descendants.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Differentiable functions
 - Functions $(a, b) \rightarrow \mathbb{R}$ differentiable at a point
 - Comparing a function and it derivative
 - Absolutely continuous functions on $[a, b]_- \leftrightarrow_- \int_{[a,-]} (L^1[a, b])$
 - Distributional derivatives
 - Fractional Sobolev spaces
 - $H^1(U)$
 - H^1_{loc}
 - $H^1_{\text{loc}}(U)$ for $U \subseteq \mathbb{R}^2$
 - Double derivative/Laplace operator on the circle
 - Fractional derivative
 - Infinite limit of derivatives
 - Space of continuous and continuously differentiable functions on $\mathbb{R}$
 - $\mathcal{C}^1([a, b], \mathbb{R})$
 - Derivative operator on $\mathcal{C}^1[a, b]$
 - $(\mathcal{C}^1([a, b], \mathbb{R}), \|\cdot\|_\infty)$
 - $\mathcal{C}^k_c((a, b), \mathbb{R})$
 - Derivative of maps $\mathbb{R}^n \rightarrow \mathbb{R}^m$
 - Zooming of a map $\mathbb{R}^n \rightarrow \mathbb{R}^m$

And it has 39 siblings.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
 - Open subsets of $\mathbb{R}^n$ with $\mathcal{C}^2$ boundary
 - Complex ODEs in $\mathbb{C}$
 - Circle packing on $\mathbb{R}^2$
 - Circle packing converges to the Riemann biholomorphism
 - Bounded connected open subsets of $\mathbb{C}^n$
 - Connected circular open subsets of $\mathbb{C}^n$
 - Continuous functions on $\mathbb{R}^d$
 - Dyadic cubes
 - Curves
 - Differentiable functions

- [Differential forms on \$\mathbb{R}^n\$](#)
- [Fourier-Wigner transform](#)
- [Harmonic functions composed with conformal maps](#)
- [Hilbert transform](#)
- [Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values](#)
- [Holomorphic subsets of \$\mathbb{C}^n\$](#)
- [Regular hypersurfaces in \$\mathbb{R}^{2n}\$](#)
- [Orientable hypersurfaces in \$\mathbb{R}^n\$](#)
- [Laplace operator on \$\mathbb{R}^n\$](#)
- [\$l^\infty\(\mathbb{R}\)\$](#)
- [Lebesgue measurable subsets of and functions on \$\mathbb{R}^n, T^n, S^n\$](#)
- [Lebesgue measure of boundary of open sets in \$\mathbb{R}^n\$](#)
- [Local \$\mathbb{R}\$ -algebras](#)
- [\$l^p\(\mathbb{R}\)\$](#)
- [Metric density of subsets of \$\mathbb{R}^n\$](#)
- [Monotone functions on \$\mathbb{R}\$](#)
- [Cauchy integral of periodic functions](#)
- [Polygons with integer vertices](#)
- [Open smooth maps \$U \subseteq \mathbb{R}^2 \rightarrow \mathbb{C}\$](#)
- [Discrete subgroups of \$\mathbb{R}^n\$](#)
- [Discrete cocompact subgroups of \$\mathbb{R}^n\$, flat tori](#)
- [Ramified germs of smooth and holomorphic functions](#)
- [Open subsets of \$\mathbb{R}^n\$](#)
- [Open subsets of \$\mathbb{R}^n\$ equipped with the flat metric](#)
- [Closed subsets of \$\mathbb{R}^n\$ equipped with smooth functions](#)
- [Quasi-analytic smooth functions on \$\mathbb{R}\$](#)
- [Star-shaped subsets of \$\mathbb{R}^n\$](#)
- [ODEs in \$\mathbb{R}^n \leftrightarrow\$ Vector fields in \$\mathbb{R}^n\$](#)