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Symplectic homogeneous manifolds

Definition. A **symplectic homogeneous manifold** is a symplectic manifold with a transitive symplectic Lie group action.

[1]

- [Ad* orbits are symplectic manifolds](#)

classification of homogeneous symplectic manifolds

Let X, G be simply connected and

$$G \curvearrowright (X, \omega)$$

by a homogeneous symplectic G -manifold. Thus it is a **almost Hamiltonian G -action**.

Now define

$$\overline{\mathfrak{g}} := \{(h, Z) \in \mathcal{C}^\infty(X) \times \mathfrak{g} \mid Z = -\omega^{-1}dh\}$$

with bracket

$$[(h_1, Z_1), (h_2, Z_2)] = [\{h_1, h_2\}, [Z_1, Z_2]]$$

then we have the exact sequence

$$\begin{array}{ccccccc} 0 & \rightarrow & \mathbb{R} & \rightarrow & \overline{\mathfrak{g}} & \rightarrow & \mathfrak{g} \rightarrow 0 \\ & & & & (h, Z) & \mapsto & Z \end{array}$$

The corresponding Lie group $\overline{G}$ we have

$$\overline{G} \rightarrow G \curvearrowright X$$

- Now define

$$\begin{array}{l} \Phi : X \rightarrow \overline{\mathfrak{g}}^* \\ x \mapsto \Phi_x : \Phi_x(h, Z) = h(x) \end{array}$$

- Then

$$\begin{array}{l} \omega(Z, -) = -dh \\ \dots \end{array}$$

so this Φ is indeed a moment map.

- By construction ...

[2]

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And it has 8 siblings.

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1. [Symplectic homogeneous space](#) - Encyclopedia of Mathematics ↔
2. [https://people.math.sc.edu/kellerlv/Symplectic_Geometry_HW1%20\(41\).pdf](https://people.math.sc.edu/kellerlv/Symplectic_Geometry_HW1%20(41).pdf) ↔