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# Image of discrete subgroups of topological groups

## projection of discrete subgroups is not discrete in general

### projection of discrete subgroup modulo a compact subgroup is discrete!

Let  $G, K$  be metrizable topological groups.

$$\begin{array}{ccc} \Gamma & \overset{\text{discrete}}{\subset} & G \times \underbrace{K}_{\text{compact}} \\ & & \downarrow \pi_1 \\ & & G \end{array}$$

Let  $\pi_1(\Gamma)$  is not discrete in  $G$ . Then there is a sequence  $\{g_n\} \subseteq \pi(\Gamma) \setminus \{i_G\}$

$$g_n \rightarrow i_G$$

Choose  $k_n \in K$  such that  $(g_n, k_n) \in \Gamma$ .

There is thus a subsequence

$$\underbrace{(g_{n_j}, k_{n_j})}_{\in \Gamma} \xrightarrow{j \rightarrow \infty} (i_G, k) \in G \times K$$

Because  $\Gamma$  is *closed* we conclude  $(i_G, k) \in \Gamma$ . But this contradicts discreteness of  $\Gamma$ . Hence,  $\pi_1(\Gamma)$  is discrete.

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