

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on October 17, 2025 5:25:13 AM, was last modified on January 29, 2026 3:36:58 PM, and was exported on September 7, 2026 3:31:20 PM.

Cut locus in a complete Riemannian manifold

Cut Points

Theorem [10.26](#) showed that once a geodesic passes its first conjugate point, it ceases to be minimizing. The converse, however, is not true: a geodesic can cease to be minimizing without reaching a conjugate point. For example, on the cylinder $\mathbb{S}^1 \times \mathbb{R}$ with the product metric, there are no conjugate points along any geodesic; but no geodesic segment that wraps more than halfway around the cylinder is minimizing (Fig. [10.10](#)).

Therefore it is useful to make the following definitions. Suppose (M, g) is a complete, connected Riemannian manifold, p is a point of M , and $v \in T_p M$. Define the *cut time of (p, v)* by

$$t_{\text{cut}}(p, v) = \sup \{ b > 0 : \text{the restriction of } \gamma_v \text{ to } [0, b] \text{ is minimizing} \},$$

Current note has 0 direct children and 0 total descendants.

- [structures based on sets](#)
 - [manifolds](#)
 - [ℝ-manifold semi-Riemannian](#)
 - [Complete Riemannian manifolds](#)
 - [Cut locus in a complete Riemannian manifold](#)

And it has 14 siblings.

- [structures based on sets](#)
 - [manifolds](#)
 - [ℝ-manifold semi-Riemannian](#)
 - [Complete Riemannian manifolds](#)
 - [Integrating sectional curvature](#)
 - [Riemannian manifolds with negative curvature](#)
 - [Riemannian manifolds with non-positive sectional curvature](#)
 - [Cut locus in a complete Riemannian manifold](#)
 - [Exponential map for a complete Riemannian manifold](#)
 - [Geodesic completeness](#) $\iff$ [metric completeness of Riemannian manifolds](#)
 - [Geodesics in homotopy classes](#)
 - [Homogeneous Riemannian manifolds](#)
 - [Homogeneous and isotropic Riemannian manifolds](#)
 - [Poles of a complete Riemannian manifold](#)
 - [Riemannian symmetric spaces](#)
 - [Uniquely geodesic complete Riemannian manifolds](#)
 - [Riemannian homogeneous manifolds where \$\exp_p\$ is a diffeomorphism](#)
 - [Complete Riemannian manifolds without conjugate points](#)