Info

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$\mathcal{C}^\infty(M)$

Definition.

$$\mathcal{C}^\infty(U) := \{f : U \to \mathbb{R} : f \text{ is smooth}\}$$

- *addition* $+ : \mathcal{C}^\infty(M) \times \mathcal{C}^\infty(M) \to \mathcal{C}^\infty(M)$

$$(f+g)(p) := f(p) + g(p)$$

- *multiplication* $\mathcal{C}^\infty(M) \times \mathcal{C}^\infty(M) \to \mathcal{C}^\infty(M)$

$$(fg)(p) := f(p)g(p)$$

$(\mathcal{C}^\infty(M), +,)$ is a commutative ring.

- f
  - derivations $\mathcal{C}^\infty(M) \to \mathbb{R}$ as vectors Tangents on smooth manifolds as derivations $\mathcal{C}^\infty(M) \to \mathbb{R}$
  - derivations $\mathcal{C}^\infty(M) \to \mathcal{C}^\infty(M)$ as vector fields Vector field on smooth manifold M as derivation of $\mathcal{C}^\infty(M)$
  - automorphisms of $\mathcal{C}^\infty(M)$ as diffeomorphisms

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Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Functions from smooth manifolds to $\mathbb{R}$
        - $\mathcal{C}^\infty(M)$

And it has 3 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Functions from smooth manifolds to $\mathbb{R}$
        - Critical values of smooth functions to $\mathbb{R}$
        - Equations for functions on smooth manifolds
        - $\mathcal{C}^\infty(M)$