Riemannian manifolds with non-positive sectional curvature *AKA* Cartan-Hadamard manifolds

(Cartan-Hadamard) Let (M, g) be a complete Riemannian manifold with non-positive sectional curvature $\kappa_g \leq 0$ then for any $p \in M$ the exponential map

$$\exp_p : T_pM \to M$$

is a covering map. In particular if M is simply connected then $\exp_p$ is a diffeomorphism.

universal cover

Proposition: Let (M, g) be a complete Riemannian manifold. Then the following are equivalent

1. every geodesic in (M, g) is (globally) **length-minimizing**
2. every unit speed geodesic in (M, g) is a minimal geodesic
3. every pair of points in (M, g) is joined by a **unique** minimal geodesic segment
4. the exponential

$$\exp_p : T_pM \to M$$

   is a diffeomorphism for all $p \in M$