Uniquely geodesic Riemannian homogeneous manifolds

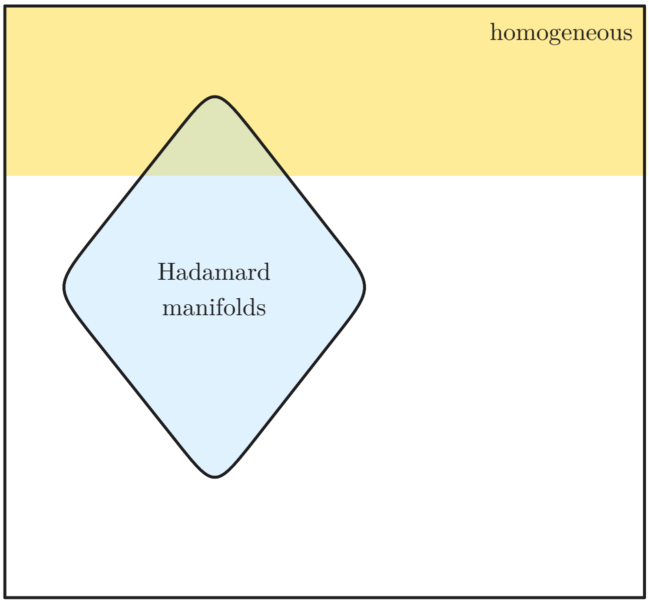
Proposition: Let (M, g) be a complete Riemannian manifold. Then the following are equivalent

1. every geodesic in (M, g) is (globally) **length-minimizing**
2. every unit speed geodesic in (M, g) is a minimal geodesic
3. every pair of points in (M, g) is joined by a **unique** minimal geodesic segment
4. the exponential

$$\exp_p : T_pM \to M$$

is a diffeomorphism for all $p \in M$

uniquely geodesic $\iff$ $\exp_p$ is a diffeo $\forall p \in M$



Simply connected, homogeneous *Cartan-Hadamard manifolds* are examples of such Riemannian manifolds. [1]

---

1. riemannian geometry - A Converse to Cartan–Hadamard theorem? - MathOverflow ↩