Exponential map from tangent bundle

Geodesic flow is a flow on TM for a Riemannian manifold (M, g) then we have a smooth map

$$\exp(-) : \mathcal{E} \subseteq TM \to TM \xrightarrow{\pi} M$$

where

$$\begin{aligned} \mathcal{E}_p &:= \{V_p \in T_pM \mid \exists \gamma_v : [0,1] \to M \text{ a geodesic starting at } V_p\} \\ \mathcal{E} &:= \bigcup_{p \in M} \mathcal{E}_p \\ \exp(V_p) &= \gamma_{Vp}(1) \end{aligned}$$