Quotients of Riemannian manifolds

(Quotients of Riemannian manifolds) Let (M, g) be a Riemannian manifold and let $\pi : M \to X$ be a surjective smooth submersion. Let

$$G \curvearrowright (M, g_M)$$

be an isometric, vertical action which is transitive on fibers of π then there is a unique Riemannian metric g_X on X such that

$$\pi : (M, g_M) \to (X, g_X)$$

is a Riemannian submersion.

Corollary: Let

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is a smooth action by isometries which is *free and proper*. Then by

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there exists an **unique** Riemannian metric $g_{\frac{M}{G}}$ on the quotient manifold $\frac{M}{G}$ such that

$$\pi : (M, g_M) \to \left(\frac{M}{G}, g_{\frac{M}{G}}\right)$$

is a **Riemannian submersion**.

Warning

The left action

$$H \curvearrowright (G, g_l)$$

is isometric, free and proper but the orbits

$$Hg$$

are **right cosets** so the quotient space is the **right coset space** ${}_H\backslash^G$ and not the standard left coset space $G/_H$.