When does a map between topological spaces send discrete sets to discrete sets

Even if X is compact, $\pi_1 : X \times X \to X$ does not send discrete sets to discrete sets.

Consider

$$\begin{array}{c} [0,1]^2 \\ \Big\downarrow{\scriptstyle \pi_1} \\ [0,1] \end{array}$$

and

$$\underbrace{\left\{\left(\frac{1}{n},\frac{1}{n}\right)\middle| n \geq 1\right\} \cup \{(0,1)\}}_{\text{discrete}} \mapsto \underbrace{\left\{\frac{1}{n}\middle| n \geq 1\right\} \cup \{0\}}_{\text{not discrete}}$$