

Weak[★] topology on dual of a topological vector space

↗ **Definition.** Weak[★] topology on dual X^\star of a topological vector space

Let X be a topological k -vector space whose dual is X^\star . Then we have the evaluation maps


$$\begin{aligned} \text{ev}_x : X^\star &\rightarrow k \\ \varphi &\mapsto \varphi(x) \end{aligned}$$

for each $x \in X$ which gives an injection

$$\begin{aligned} X &\hookrightarrow X^{\star\star} \\ x &\mapsto \text{ev}_x \end{aligned}$$

Assume these functionals separates points of X^\star . The X -topology^[1] on X^\star is called the **weak[★]-topology** on X^\star under which topology

$$X^{\star\star} \cong X$$

1.  Let X be a k -vector space and $V \leq X^*$ be a vector space of separating linear functionals on X . Then the V -topology \mathfrak{T}_V makes X into a locally convex space such that

$$X^\star \cong V$$

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