Topological vector space over $\mathbb{R}$ or $\mathbb{C}$

Definition. Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$

A vector space X over $k \in \{\mathbb{R}, \mathbb{C}\}$ is a **topological k-vector space** if there is a T_1 topology on X such that

$$+ : X \times X \to X$$
$$k \times X \to X$$

are continuous.

Definition. Vector bounded subsets of topological vector spaces

A subset E of a topological vector space is **vector bounded** if for every nb V of $0_X \in X$ there is a $s(V) > 0$ such that

$$\forall t > s(V), E \subseteq tV$$

Proposition: Let X be a topological vector space. Then translations and scaling are homeomorphisms of X onto X.

Definition. Types of topological vector spaces

X is	if
locally convex	there is a local base of convex subsets
locally vector bounded	0_X has a bounded onb
locally compact	0_X has a onb whose closure is compact
metrizable	there is a metric d which induces the topology on X
F-space	there is an translation invariant complete metric d which induces the topology on X
Frechet space	if X is a locally convex F-space
normable	there exists a norm which induces the topology on X
vector Heine-Borel	every closed and vector bounded subset of X is compact