# Finitely generated groups

Finitely generated groups $\Gamma$ are (equivalently)

- generated

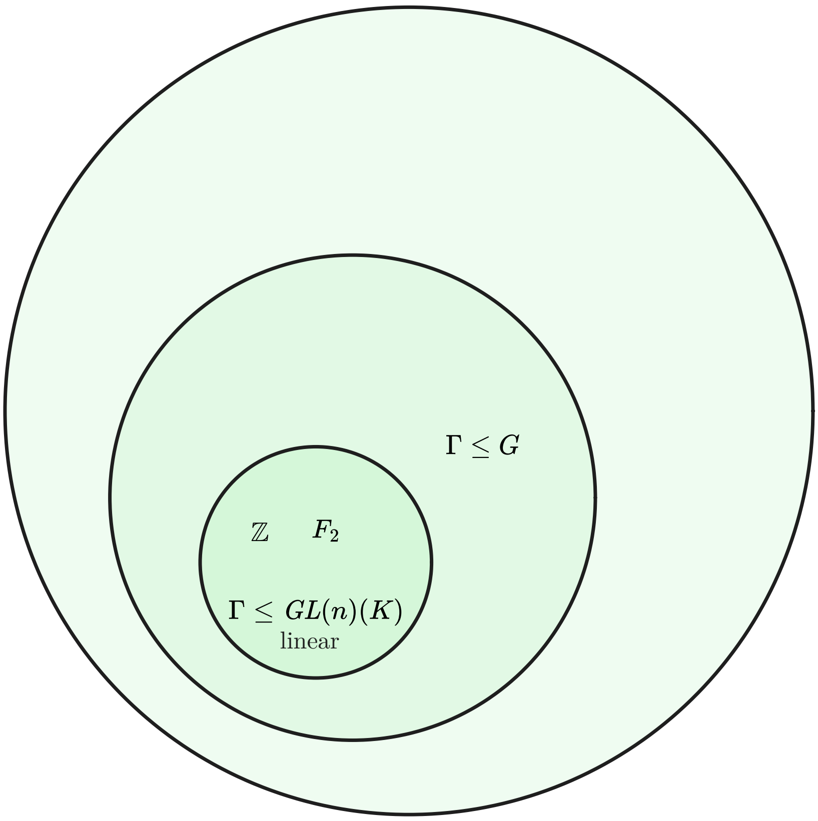
$$\Gamma = \langle \gamma_1, \ldots, \gamma_m \rangle$$

  by finitely many $\gamma_1, \ldots, \gamma_m \in \Gamma$
- quotients of free groups

$$F_m \twoheadrightarrow \Gamma$$

- have a minimum, finite size of generating set

$$r(\Gamma) := \min \{|S| | S \subseteq \Gamma, \langle S \rangle = \Gamma\} < \infty$$




They have the following (weaker) properties

- $\Gamma$ is countable
- $\mathrm{Hom}\mathbf{Grp}(\Gamma, G)$ is finite for any finite group $G$
  - has cardinality $\leq |G|^{r(\Gamma)}$

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- quasi-geometry
  - growth functions

- Ends
- boundary

- Amenable groups
- actually/virtually [p] groups
  - Abelian
  - free
  - nilpotent