Cross ratios and Mobius maps between metric spaces

Definition. Cross ratios and Mobius maps between metric spaces

Let (X, d) be a metric space. The **cross-ratio** of four pairwise distinct points $x, y, z, w \in X$ is

$$[x, y, z, w]_d := \frac{d(x, z)d(y, w)}{d(x, w)d(y, z)}$$

A map $f : X \to Y$ between metric spaces $(X, d_X), (Y, d_Y)$ is

- **Mobius** if it preserves cross-ratios

$$\forall x, y, z, w \in X, [x, y, z, w]_{d_X} = [f(x), f(y), f(z), f(w)]_{d_Y}$$

- **quasi-Mobius** if

$$\forall x, y, z, w \in X, [x, y, z, w]_{d_X} = \eta([f(x), f(y), f(z), f(w)]_{d_Y})$$

  for some homeomorphism $\eta : [0, \infty) \to [0, \infty)$.

(Efremovich-Tihomirova) Let X, Y be proper δ-hyperbolic spaces and their boundaries $\partial X, \partial Y$ be equipped with visual metrics. Then every quasi-isometry $X \to Y$ extends to a quasi-Mobius homeomorphism $\partial X \to \partial Y$

$$\mathrm{HomMet}_{\mathsf{preQ}}(X, Y) \to \mathrm{MobQ}(\partial X, \partial Y)$$

(Moreover the distortion function η only depends on δ and the quasi...)

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1. Mostow type rigidity theorems - Mostow.pdf ↩