Info

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$\exp\left(\frac{z+1}{z-1}\right)$

Quote

K. Hoffman called this the "world's greatest function"!

> **Proposition:**
>
> $$\exp\left(\frac{z+1}{z-1}\right) : \mathbb{C} \setminus \{1\} \to \mathbb{C}$$
>
> is holomorphic and
>
> $$\left|\exp\left(\frac{z+1}{z-1}\right)\right| = \exp \operatorname{Re}\left(\frac{z+1}{z-1}\right) = \exp\left(\frac{|z|^2-1}{|z-1|^2}\right)$$

- $$e^{x+iy} = e^x(\cos y + i \sin y)$$

  and

  $$\frac{z+1}{z-1} = \frac{\overbrace{(z+1)^2}^{(x+1+iy)^2}}{|z-1|^2} = \frac{(x+1)^2 - y^2 + 2i(x+1)y}{(x-1)^2+y^2}$$

So on $z \in \mathbb{D}$ we have

$$\exp\left(\underbrace{\frac{|z|^2-1}{|z-1|^2}}_{\leq 0 \text{ on } |z|<1}\right) \leq 1$$

[1]

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - $\exp\left(\frac{z+1}{z-1}\right)$

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $$\cos\sqrt{z}$$
    - The complex logarithm
    - $\sum_{n\geq 0} z^{n!}$
    - $z^3 - 3z$
    - $$\frac{r_0}{1+\epsilon\cos b\theta}$$
    - $\cos\left(\pi\frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - e^{-x^2}
    - [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.$]]
    - $\int \frac{1}{x^2+bx+c}$
    - $\frac{\exp(z)}{1-z}$
    - $\sin x$
    - $\sin(x\sin(x))$
    - $\frac{\sin x}{x}$
    - stock.sin x by xk
    - $\sin x^2$
    - $\sqrt{z}$
    - $\sqrt{z^2-1}$
    - Theta functions
    - Weierstrass elliptic function $\wp$
    - Weierstrass' primary factors
    - $x^2 \sin\frac{1}{x}$
    - $x + x^2\sin\frac{1}{x}$

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1. Marshall - Complex Analysis.pdf > page=54 ↩