Sobolev space $H^1(U)$

Definition. **Sobolev space $H^1(U)$**

Let $U(\text{open}) \subseteq \mathbb{R}^n$ be an open set. Then the **Sobolev space** of $L^2(U)$ functions with distributional derivatives in $L^2(U)$

$$H^1(U) := \left\{ f \in L^2(U) \middle| \forall 1 \le j \le n, \overset{\int}{\partial}_j f \in L^2(U) \right\}$$

is equipped with the inner product

$$\langle f, g \rangle_{H^1} := \langle f, g \rangle_2 + \sum_{1 \le j \le n} \left\langle \overset{\int}{\partial}_j f, \overset{\int}{\partial}_j g \right\rangle_2$$

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is a separable Hilbert space.

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