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Ad* orbits are symplectic manifolds

Definition. Symplectic structure on coadjoint orbits of Lie groups

Let

$$\Omega \subset \mathfrak{g}^*$$

be a coadjoint orbit $\text{Ad}^* : G \curvearrowright \mathfrak{g}^*$ of a Lie group G . Let $F \in \Omega$.

- By definition Ω is a transitive (left) G -space with stabilizer stabilizer of F being $\text{Ad}_{G,F}^*$ (a closed subgroup).
- Thus the surjective smooth map

$$\begin{aligned} G &\rightarrow \Omega \\ g &\mapsto \text{Ad}_g^*(F) \end{aligned}$$

with descends to bijection

$$\frac{G}{\text{Ad}_{G,F}^*} \rightarrow \Omega$$

giving a smooth structure on the orbit Ω making it a *smooth immersed manifold* in $\mathfrak{g}^*$. We construct a anti-symmetric bilinear map

$$\begin{aligned} \omega_F : \frac{\mathfrak{g}}{\text{Lie}(\text{Ad}_{G,F}^*)} \times \frac{\mathfrak{g}}{\text{Lie}(\text{Ad}_{G,F}^*)} &\rightarrow \mathbb{R} \\ (X, Y) &\mapsto F([X, Y]) \end{aligned}$$

Proposition:

- The bilinear map

$$\begin{aligned} \omega_F : \frac{\mathfrak{g}}{\text{Lie}(\text{stab}_G(F))} \times \frac{\mathfrak{g}}{\text{Lie}(\text{stab}_G(F))} &\rightarrow \mathbb{R} \\ (X, Y) &\mapsto F([X, Y]) \end{aligned}$$

defines a Ad^* -invariant **symplectic form** on the orbit $\Omega \subset \mathfrak{g}$.

- The coadjoint action $G \curvearrowright \Omega$ is **Hamiltonian** with moment map

$$\iota : \Omega \hookrightarrow \mathfrak{g}^*$$

Thus

$$\text{Ad}^* : G \curvearrowright (\Omega, \omega) \subset \mathfrak{g}^*$$

is a **homogeneous Hamiltonian G -manifold**.

[1]

[2]

Lie group	coadjoint orbits
$SO(3, \mathbb{R}), SU(2, \mathbb{C})$	$\{0\}, S^2$
compact	Kahler ^[3]
nilpotent	?

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 - [ℝ-manifold_group_\(Lie_groups\)](#)
 - [Adjoint representation of a Lie group](#)
 - [Ad* orbits are symplectic manifolds](#)
 - [Adjoint_group_of_a_Lie_group](#)

1. page 48 of the pdf

[Matthias Peter - The Orbit Method for Compact Connected Lie Groups](#)



2. [studenttheses.uu.nl/bitstream/handle/20.500.12932/7207/Maes%2CJMA2011.pdf?sequence=1&isAllowed=y#page=40.00](#)

3. prop 3.1.21 of the pdf

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