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Curvature of a Riemannian manifold

Definition. Riemann curvature tensor of a Riemannian manifold

The assignment

$$(X, Y, Z) \mapsto ([\nabla_X, \nabla_Y] - \nabla_{[X, Y]})Z$$

produces a (1,3)-tensor on M .

The **Riemann curvature tensor** is the (0,4)-tensor

$$R(X, Y, Z, W) := g([\nabla_X, \nabla_Y] - \nabla_{[X, Y]})Z, W)$$

The Riemann curvature (0,4)-tensor is actually

$$R \in \Gamma \mathbf{Sym}^2 \mathbf{\Lambda}^2 TM$$

and it satisfies the two **Branchi identities**

$$R(W, X, Y, Z) + R(X, Y, W, Z) + R(Y, W, X, Z) = 0$$

Definition. Ricci and scalar tensors

$$\text{Ric}(X, Y) := \text{tr}(Z \mapsto$$

		R^l_{ijk}
Riemann	$R : \mathbf{Sym}^2 \mathbf{\Lambda}^2 TM \rightarrow \mathbb{R}$	$R_{ijkl} = g_{lm} R^m_{ijk}$
	we may think of it as a <i>symmetric bilinear form on the 2-wedge tangent space</i> $\mathbf{\Lambda}^2 TM \times \mathbf{\Lambda}^2 TM \rightarrow \mathbf{Sym}^2 \mathbf{\Lambda}^2$ $(X \wedge Y, Z \wedge W) \mapsto R(X, Y, Z, W)$	
	and consider the <i>quadratic form</i> of the above bilinear form $\mathbf{\Lambda}^2 TM \rightarrow \mathbb{R}$ $R(X \wedge Y) \mapsto R(X, Y, X, Y)$	
sectional	$\kappa : \mathbf{Gr}^2 TM \rightarrow \mathbb{R}$ $\kappa(\mathbb{R}\langle v, w \rangle) := \frac{R(v, w, w, v)}{\ v \wedge w\ _g^2}$	
	$(X, Y) \mapsto \text{tr}(Z \mapsto R(X, Z)Y)$	$g^{pq} R_{ipjq}$
Ricci	$\text{Ric} : T^u(M, g) \rightarrow \mathbb{R}$	
scalar	$\text{sc} := \text{tr}(\text{Ric}) : M \rightarrow \mathbb{R}$	$\kappa = R^i_i = g^{ij} R_{ij}$
total scalar curvature	$\int_M \text{sc}_g \lambda_g$	

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