Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on February 9, 2026 4:46:34 PM,
was last modified on August 20, 2026 5:42:26 PM, and
was exported on September 7, 2026 3:09:52 PM.

$\mathcal{C}(U, X)$

Definition. $\mathcal{C}(U, X)$

Let (X, d) be a complete metric space, $U \subseteq \mathbb{R}^n$ be a open subset and

$$\mathbf{EndTop}(U, X)$$

be the space of all continuous functions $U \to X$.

- For $K \subseteq U$ compact we have the distance

$$\rho_K(f, g) := \sup \{d(f(z), g(z)) | z \in K\}$$

  for all $f, g \in \mathcal{C}(G, X)$.
- Then for $U = \bigcup_{n \in \mathbb{N}} K_n$ where K_n is compact and $K_n \subseteq \operatorname{int}(K_{n+1})$ we define the metric

$$\rho(f, g) := \sum_{n \in \mathbb{N}} \frac{1}{2^n} \frac{\rho_{K_n}(f, g)}{1 + \rho_{K_n}(f, g)}$$

  on $\mathbf{EndTop}(U, X)$.

The (metrizable) topological space $\mathbf{EndTop}(U, X)$ with the topology induced by the metric ρ is denoted $\mathcal{C}(U, X)$.

Proposition: Consider the space equipped with the distance

$$(\mathbf{EndTop}(U, X), \rho)$$

- ρ is a complete metric

- the topology is given by

  Definition. Compact-open topology on Y^X

  Let X be a topological space and (Y, d) be a metric space. Let $K \subseteq X$ be compact and $\epsilon > 0$ and consider the subsets

$$B_\epsilon(f, K) := \left\{ g \in Y^X \middle| \sup_{x \in K} d(f(x), g(x)) < \epsilon \right\}$$

  The topology on Y^X generated by these subsets is the **compact-open** topology $\mathfrak{T}_{\text{cpt}}$.

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Continuous functions on $\mathbb{R}^d$
      - $\mathcal{C}(U, X)$

And it has 5 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Continuous functions on $\mathbb{R}^d$
      - $\mathcal{C}_0[0, 1]$
      - Continuous functions on $\mathbb{R}$ with infinite limit are uniformly continuous
      - Intermedia value property of continuous functions on intervals
      - $\mathcal{C}(U, X)$
      - $\mathcal{C}[0, 1]$