Info

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Manifold with a flow

Definition. Manifold with a smooth flow

A **manifold with a smooth flow** is a pair

$$(M, X)$$

where M is a smooth manifold and $X \in \mathrm{Vec}(M)$.
A *morphism* of two manifolds with flows

$$\phi : (M_1, X_1) \to (M_2, X_2)$$

is a smooth map $\phi : M_1 \to M_2$ such that $\phi \circ \exp(tX_1) = \exp(tX_2) \circ \phi$ or equivalently the two vector fields are ϕ-related

$$\mathrm{d}_y\phi(X_y) = X_{\phi(y)}$$

This gives us a category of manifolds with a smooth flow

FlowMan

check if two conditions above are equivalent

class	
Trivial	$(M, 0)$
Linear flows	$(\mathbb{R}^n, A)$ where $A \in \mathrm{Mat}(n, \mathbb{R})$
Geodesic flows of (M, g)	$(TM, \exp)$ where (M, g) is a semi-Riemannian manifold and exp is the geodesic flow
Hamiltonian flows on (M, ω)	$(M, \omega, -\omega^{-1}\mathrm{d}H)$ where (M, ω) is a symplectic manifold and $H \in \mathcal{C}^\infty(M)$
Lagrangian flows	$(TM, \mathcal{E}_L)$ where $\mathcal{E}_L$ is the Euler-Lagrange vector field of $L \in \mathcal{C}^\infty(TM)$
flow of a mechanical manifold $(M, g, \mathcal{F})$	
flow of a *conservative* mechanical manifold $(M, g, -\mathrm{d}U)$	
Isometric flows	(M, g, X) such that $\mathcal{L}_X g = 0$
Suspension of diffeomorphisms of smooth manifolds	• Functor? $$\begin{array}{ccc} \mathrm{End}(\mathrm{Man}) & \to & \mathrm{FlowMan} \\ \begin{array}{c} M \\ \downarrow f \\ M \end{array} & \mapsto & (M_f, \frac{\partial}{\partial t}) \end{array}$$

Definition. Immersion and embeddings of manifolds with a smooth flow

An **immersion/embedding** of (M_1, X_1) into (M_2, X_2) is a morphism

$$\phi : (M_1, X_1) \to (M_2, X_2)$$

where ϕ is an injective immersion/embedding.

	immersions
stationary solution	$(\{\bullet\}, 0) \hookrightarrow (M, X)$
periodic solution	$(\mathbb{R}/\mathbb{Z}, 1) \hookrightarrow (M, X)$
invariant torus	$((\mathbb{R}/\mathbb{Z})^d, \alpha) \hookrightarrow (M, X)$

Definition. Lorentz flows

$$(M, g, F)$$

Let (M, g) be a semi-Riemannian manifold and

$$F \in \Omega^2(M), \mathrm{d}F = 0$$

then by the symplectic structure on T^*M

$$\omega^{qF} := \mathrm{d}\theta + q\pi^*_{T^*M}F$$

we have the ω^{qF}-Hamiltonian vector field of the smooth function

$$\begin{aligned} T^*M &\to \mathbb{R} \\ p &\mapsto \frac{\|p\|_g^2}{2} \end{aligned}$$

Current note has 1 direct children and 2 total descendants.

- structures based on sets
    - manifolds
        - Manifold with a flow
            - Semi-Riemannian mechanical manifold
                - Potential mechanical manifold M^N

And it has 20 siblings.

- structures based on sets
    - manifolds
        - Closed aspherical manifolds
        - $\mathbb{C}$-manifold
        - Hermitian manifolds
        - Kahler manifolds
        - Kahler manifolds of complex dimension 2, AKA Kahler surfaces
        - smooth $\mathbb{R}$-manifold
        - Principal bundles over smooth manifolds
        - Manifold with a flow
        - $\mathbb{R}$-manifold group (Lie groups)
        - Homogeneous manifold
        - Finsler manifold
        - $\mathbb{R}$-manifold semi-Riemannian
        - Sub-Riemannian manifolds
        - $\mathbb{R}$-manifold Poisson
        - $\mathbb{R}$-manifold symplectic
        - Multisymplectic manifolds
        - Spin $\mathbb{R}$-manifold
        - $\mathbb{R}$-manifold topological
        - Toric manifolds
        - Manifolds with compatible triples