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Existence and uniqueness of $(\mathbb{R}, +, \times, <)$

 **(Existence of $\mathbb{R}$)** There exists an unique ordered field $\mathbb{R}$ with the sup property upto isomorphism. Moreover, $\mathbb{R}$ contains $\mathbb{Q}$ has a subfield, and is its metric completion under the standard norm.

Construction of the reals

- Dedekind cuts in $\mathbb{Q}$
- Equivalence classes of Cauchy squences in $\mathbb{Q}$

Uniqueness

$$f : \mathbb{R} \rightarrow \mathbb{F}$$

but

$$j : \mathbb{Q} \rightarrow \mathbb{F}$$

is an injective field homomorphism and

$$f|_{\mathbb{Q}}(r) = j(r)$$

for any $\alpha \in \mathbb{R}$,

$$f(\alpha) := \sup\{j(r) : r \in \mathbb{Q}, r < \alpha\}$$

Current note has 0 direct children and 0 total descendants.

- [stretchy](#)
 - $(\mathbb{R}, +, \times, <, \sup)$
 - [Existence of \$\mathbb{R}\$](#)

And it has 3 siblings.

- [stretchy](#)
 - $(\mathbb{R}, +, \times, <, \sup)$
 - [Connected sets in \$\mathbb{R}\$](#)
 - [Existence of \$\mathbb{R}\$](#)
 - [Metrics on \$\mathbb{R}\$](#)