Info

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Locally compact and 2-countable Hausdorff topological spaces are paracompact with *countable refinement*

Definition. A topological space is **paracompact** if any open covering admits a locally finite refinement.

finding a base of opens with compact closure

Locally compact, 2-countable Hausdorff space $\implies \exists$ a countable base of open sets with compact closure

Let X be locally compact Hausdorff space. Let B be a countable basis of X.

- $\forall x \in X, \exists$ open $N_x \ni x$ which has compact closure.
- Now some $V \in B$ contains x and is contained in N_x.
- closure of V is a closed subset of compact closure of N_x. so $\mathrm{closure}(V)$ is also compact

Hence we define a basis

$$\begin{aligned} B^* &= \{V \in B : V \subset N_x \text{ for some } x \in X\} \\ &= \{V_1, V_2 \ldots\} \end{aligned}$$

which is countable base of open sets with compact closure.

covering by increasing sets

To find a sequence of compact subsets such that

$$\begin{gathered} K_{i-i} \subset \mathrm{int}(K_i) \\ \bigcup_i K_i = X \end{gathered}$$

we take

$$K_1 = \mathrm{closure}(V_1)$$

we define the sequence by induction

covering

Let $\{W_\alpha\}_{\alpha \in I}$ be an open covering of X. For each $i \in \mathbb{N}$ we can cover

$$K_i \setminus \mathrm{int}(K_{i-1})$$

by a finite number of W_α

[1]

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement

And it has 29 siblings.

- structures based on sets
  - Topological spaces
    - Group action on topological spaces
    - Discrete subgroup acting on coset space
    - Basis and subbasis for a topology
    - Principal G-bundles over topological spaces

      $$G \curvearrowright P \to X$$
    - Vector bundles over topological spaces
    - Closure of a subset of topological space
    - Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement
    - Covering dimension of topological spaces
    - $\mathrm{EndTop}(X, Y)$
    - Eilenberg–Steenrod functors

      $$(H_n : \mathsf{Top} \times \mathsf{Top} \to \mathsf{Ab}, \partial_n)$$
    - Local fields
    - Local fields of characteristic 0
    - Topological groups
    - Continuous group action on a topological space
    - Intersection of open, dense subsets of a topological space
    - Local topological spaces
    - Covering space of a topological space

- Borel measures on a topological space
- Convergence for nets in a topological space
- Loops and a open cover of a topological space
- Paths in a topological space
- Product of topological spaces
- Ringed topological spaces
- Sheaf on open sets of a topological space
- Closed subsets of a topological space
- Discrete subsets of topological spaces
- Types of topological spaces
- Locally compact Hausdorff spaces
- Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$

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1. Vinicius Ramos-Differential Topology.L02 ↩