Info

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Discrete subsets of topological spaces

Definition. Discrete subsets of topological spaces

Let X be a topological space. Then a subset $S \subseteq X$ is **discrete** if

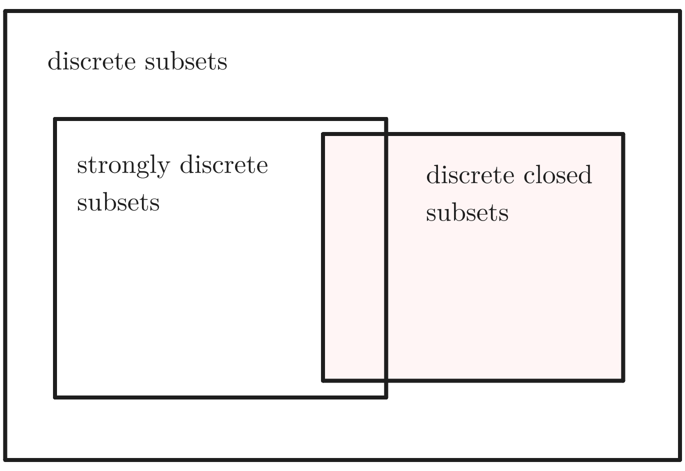
- when equipped with the subspace topology, it is a discrete topological space, that is, every subset is open
- $\iff$ for each $a \in S$ there is a onb $U_a \subseteq X$ of a such that $U_a \cap S = \{a\}$.

Definition. Strongly discrete subsets of topological spaces

Let X be a topological space. Then a subset $S \subseteq X$ is **strongly discrete** if there is a collection $\{U_a\}_{a \in S}$ of pairwise disjoint open sets such that

$$U_a \cap S = \{a\}$$

for each $a \in S$.



in metrizable spaces, discrete $\implies$ strongly discrete

Let $S \subseteq X$ be such that there is a collection $\{U_a\}_{a \in S}$ of open sets such that for all $a \in S$

$$U_a \cap S = \{a\}$$

(which makes S a discrete subset). Then there is a pairwise disjoint collection $\{V_a\}$ of subsets $V_a \subseteq U_a$ such that $V_a \cap S = \{a\}$ (making S strongly discrete).

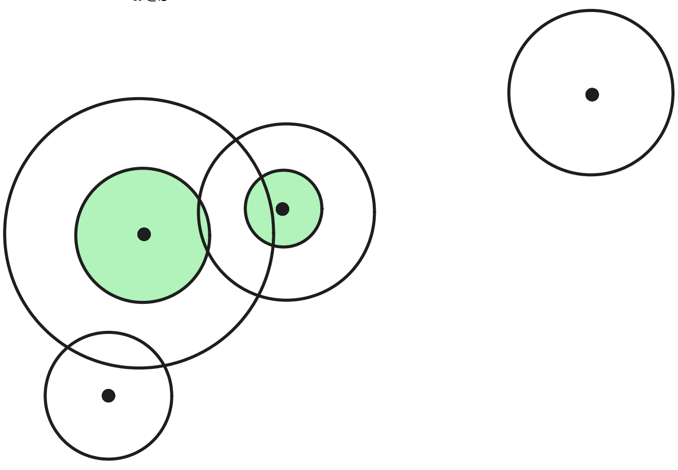
Choose $\epsilon(a) > 0$ such that $B_{\epsilon(a)}(a) \subseteq U_a$, which means

$$B_{\epsilon(a)} \cap S = \{a\}$$

- This means for $a, b \in S$

$$\begin{aligned} a \neq b \iff & d(a,b) \geq \epsilon(a), \epsilon(b) \\ \iff & d(a,b) \geq \max\{\epsilon(a), \epsilon(b)\} \\ & \implies \\ d(a,b) < \max\{\epsilon(a), \epsilon(b)\} & \implies a = b \end{aligned}$$

- Now consider $\left\{B_{\epsilon(a)/3}(a)\right\}_{a \in S}$



then

$$\begin{aligned} & y \in B_{\epsilon(a)/3}(a) \cap B_{\epsilon(b)/3}(b) \\ \implies & d(y,a) \leq \frac{\epsilon(a)}{3}, d(y,b) \leq \frac{\epsilon(b)}{3} \\ \implies & d(a,b) \leq d(a,y) + d(y,b) \\ & \leq \frac{1}{3}(\epsilon(a) + \epsilon(b)) \\ & \leq \frac{2}{3} \max\{\epsilon(a), \epsilon(b)\} \\ & < \max\{\epsilon(a), \epsilon(b)\} \\ \implies & a = b \end{aligned}$$

in Hausdorff spaces, finite subsets are strongly discrete

discrete closed subset S and locally finite collection of singletons $\{\{x\} | x \in S\}$

Current note has 0 direct children and 0 total descendants.

And it has 29 siblings.

- structures based on sets
  - Topological spaces
    - Group action on topological spaces
    - Discrete subgroup acting on coset space
    - Basis and subbasis for a topology
    - Principal G-bundles over topological spaces

      $$G \curvearrowright P \to X$$

    - Vector bundles over topological spaces
    - Closure of a subset of topological space
    - Locally compact and 2-countable Hausdorff topological spaces are paracompact with countable refinement
    - Covering dimension of topological spaces
    - $\mathrm{End}\mathsf{Top}(X, Y)$
    - Eilenberg–Steenrod functors

      $$(H_n : \mathsf{Top} \times \mathsf{Top} \to \mathsf{Ab}, \partial_n)$$

    - Local fields
    - Local fields of characteristic 0
    - Topological groups
    - Continuous group action on a topological space
    - Intersection of open, dense subsets of a topological space
    - Local topological spaces
    - Covering space of a topological space
    - Borel measures on a topological space
    - Convergence for nets in a topological space
    - Loops and a open cover of a topological space
    - Paths in a topological space
    - Product of topological spaces
    - Ringed topological spaces
    - Sheaf on open sets of a topological space
    - Closed subsets of a topological space
    - Discrete subsets of topological spaces
    - Types of topological spaces
    - Locally compact Hausdorff spaces
    - Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$