Info

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Derivative operator on $\mathcal{C}^1[a,b]$

Definition. The derivative operator

$$\begin{aligned} \mathfrak{D} : \mathcal{C}^1[a,b] &\to \mathcal{C}[a,b] \\ f &\mapsto f' \end{aligned}$$

In the sum norm

$$\mathfrak{D} : (\mathcal{C}^1[a,b], \|\ \|_\infty^1) \to (\mathcal{C}[a,b], \|\ \|_\infty)$$

we have

$$\|\mathfrak{D}(f)\|_\infty = \|f'\|_\infty \leq \|f\|_\infty + \|f'\|_\infty$$

hence $\mathfrak{D}$ is **bounded** and

$$\|\mathfrak{D}\| \leq 1$$

However, for the sup norm it is **not bounded**.

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Space of continuous and continuously differentiable functions on $\mathbb{R}$
        - $\mathcal{C}^1([a,b],\mathbb{R})$
          - Derivative operator on $\mathcal{C}^1[a,b]$

And it has 2 siblings.

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          - Derivative operator on $\mathcal{C}^1[a,b]$
          - $(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty)$