Info

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Bessel functions

Definition.

$$\mathbb{C} \to \mathbb{C}$$
$$z \mapsto \int_{s\in(-1,1)} (1-s^2)^{\nu-\frac{1}{2}} \exp(isz)$$

$$\hat{m}_{S^{n-1}}(\xi) = \int_{\omega\in S^{n-1}} \exp(i\xi\cdot\omega)\mathrm{d}m_{S^{n-1}}$$

$$\psi_n : \mathbb{R} \to \mathbb{C}$$
$$\psi_n(r) = m_{S^{n-2}}(S^{n-2}) \int_{s\in(-1,1)} (1-s^2)^{\frac{n}{2}-\frac{3}{2}} \exp(irs)$$

Asymptotics of the Bessel function

> **Lemma:** For $\alpha > -\frac{1}{2}$ we have
>
> $$\left|\int_{s\in(-1,1)} (1-s^2)^{\alpha-\frac{1}{2}} \exp(its)\right| \lesssim \frac{1}{t^{(\alpha+\frac{1}{2})}}$$
>
> as $t \to \infty$.

Assume $-1 < \alpha - \frac{1}{2} < 0$ and split

$$\int_{(-1,1)} (1-s^2)^{\alpha-\frac{1}{2}} \exp(its)\mathrm{d}s$$
$$= \int_{(0,1)} (1-s^2)^{\alpha-\frac{1}{2}} \exp(its)\mathrm{d}s +$$

- $$\int_{(0,1)} (1-s^2)^{\alpha-\frac{1}{2}} \exp(its)\mathrm{d}s =$$

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - Bessel functions

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $\cos\sqrt{z}$
    - The complex logarithm
    - $\sum_{n\geq 0} z^{n!}$
    - $z^3 - 3z$
    - $\frac{r_0}{1+\epsilon\cos b\theta}$
    - $\cos\left(\pi\frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - e^{-x^2}
    - [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.]]$
    - $\int \frac{1}{x^2+bx+c}$
    - $\frac{\exp(z)}{1-z}$
    - $\sin x$
    - $\sin(x\sin(x))$
    - $\frac{\sin x}{x}$
    - stock.sin x by xk
    - $\sin x^2$
    - $\sqrt{z}$
    - $\sqrt{z^2-1}$
    - Theta functions
    - Weierstrass elliptic function $\wp$
    - Weierstrass' primary factors
    - $x^2\sin\frac{1}{x}$
    - $x + x^2\sin\frac{1}{x}$