> ⓘ Info
>
> This note, that was found here, as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
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$\sqrt{z}$

#function/complex/1

square root on the universal cover of $\mathbb{C} \setminus \{0\}$

The inverse, or the square root

$$\sqrt{(z, \theta)} := \exp\left(\frac{1}{2}\log(z, \theta)\right)$$

is defined on the $\mathbb{Z}$-sheeted cover of punctured plane $\mathbb{R}^2 \setminus \{(0, 0)\}$.

square root on the 2-sheeted cover of $\mathbb{C} \setminus \{0\}$

> **Proposition:** The holomorphic function
>
> $$\begin{aligned} \mathbb{C} &\to \mathbb{C} \\ z &\mapsto z^2 \end{aligned}$$
>
> is a branched covering map with branched point $\{0\}$. Thus it is a covering map if we remove $0 \mapsto 0$
>
> $$\mathbb{C} \setminus \{0\} \to \mathbb{C} \setminus \{0\}$$

It is which is invariant for $\theta \mapsto \theta + 2(2\pi n)$

$$\exp\left(\frac{1}{2}\log((z, \theta) + (1, 2a))\right) = \exp\left(\frac{1}{2}\log(z, \theta) + \frac{2(2\pi n)i}{2}\right)$$

so it's 2-periodic, thus we may push this map onto the 2-sheeted cover. This is the Riemann surface for $\sqrt{z}$.

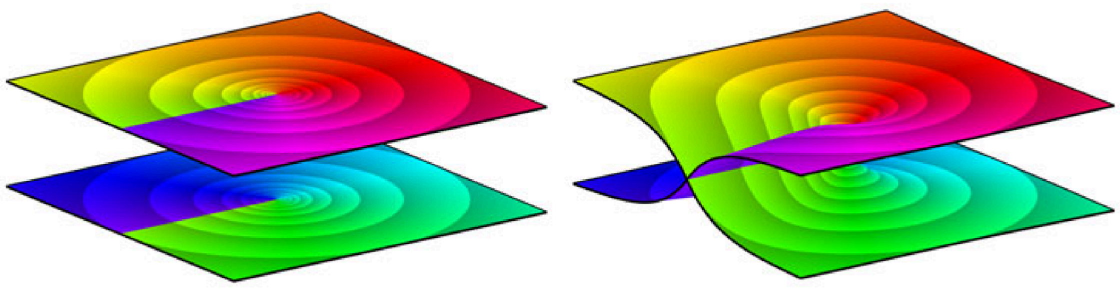
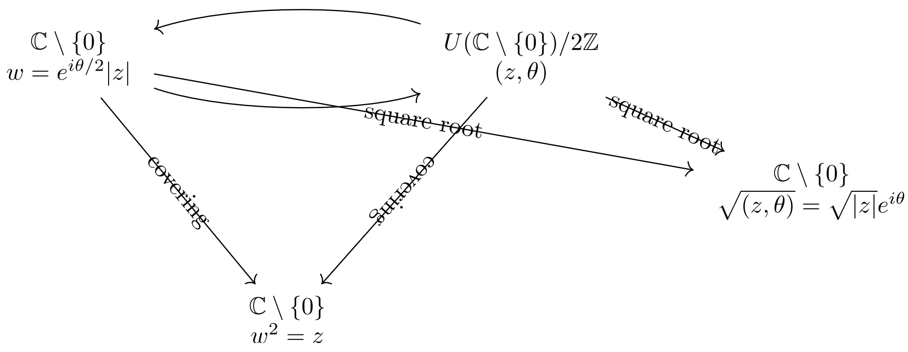


Figure 7.2: Building the concrete Riemann surface of the square root

The covering projection

$$\frac{U(\mathbb{C} \setminus \{0\})}{2\mathbb{Z}} \to \mathbb{C} \setminus \{0\}$$

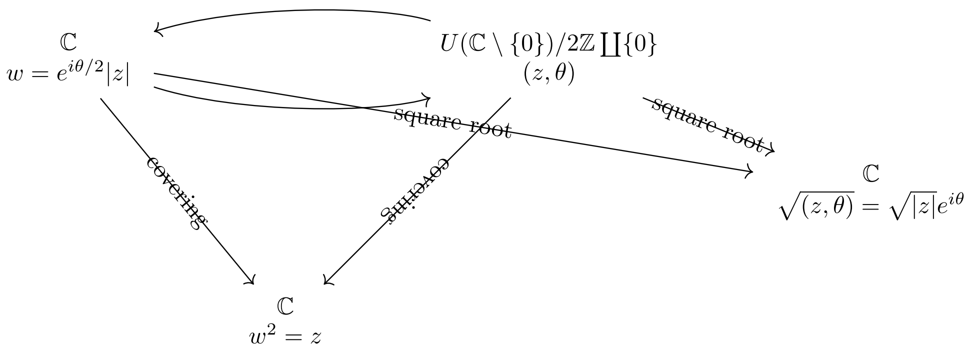
is *biholomorphically conjugate* to the z^2 covering map



More explicitly, the 2-sheeted cover is constructed by taking two *branches* for $\sqrt{z}$.

> $$\begin{aligned} \mathbb{C} &\to \frac{U(\mathbb{C} \setminus \{0\})}{2\mathbb{Z}} \\ |z|e^{i\theta/2} &\mapsto (z, \theta) \end{aligned}$$
>
> is a **biholomorphism** between the covers.

square root on the 2-sheeted cover of $\mathbb{C}$ branched at $\{0\}$



square root on the 2-sheeted cover of $\mathbb{C}P^1$ branched at $\{0, \infty\}$

The *meromorphic* function

$$\begin{aligned} \mathbb{C}P^1 &\to \mathbb{C}P^1 \\ z &\mapsto z^2 \end{aligned}$$

where $\infty \mapsto \infty$ as usual is a branched covering with branched points $\{0, \infty\}$.

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - $\sqrt{z}$

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $$\cos\sqrt{z}$$
    - The complex logarithm
    - $\sum_{n\geq 0} z^{n!}$
    - $z^3 - 3z$
    - $$\frac{r_0}{1+\epsilon\cos b\theta}$$
    - $\cos\left(\pi\frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - e^{-x^2}
    - [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.$]]
    - $\int\frac{1}{x^2+bx+c}$
    - $\frac{\exp(z)}{1-z}$
    - $\sin x$
    - $\sin(x\sin(x))$
    - $\frac{\sin x}{x}$
    - stock.sin x by xk
    - $\sin x^2$
    - $\sqrt{z}$
    - $\sqrt{z^2-1}$
    - Theta functions
    - Weierstrass elliptic function $\wp$
    - Weierstrass' primary factors
    - $x^2\sin\frac{1}{x}$
    - $x + x^2\sin\frac{1}{x}$