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# Semi-Riemannian mechanical manifold

Definition. Mechanical Riemannian manifold

A **mechanical Riemannian manifold** is a triple $(M, g, \mathcal{F})$ where $(M, g)$ is a Riemannian manifold called *configuration space* and

$$\mathcal{F} : TM \to T^*M$$

is a smooth vector bundle morphism called *external force*.
This defines a *manifold with a flow*

$$(TM, X)$$

where

$$X_{p,v} \ldots ?$$

whose integral curves when projected onto $M$ satisfies

$$g(\Delta_{\dot{c}}\dot{c}, -) = \mathcal{F}(\dot{c})$$

The external force (and the mechanical manifold) is said to be

- **positional** if $F(v)$ only depends on $\pi(v)$
- **conservative** if there exists $U \in \mathcal{C}^\infty(M)$ such that $\mathcal{F}(v) = -\mathrm{d}U_{\pi(v)}$ for all $v \in TM$ and in this case $U$ is called a *potential energy* for $\mathcal{F}$

The **kinetic energy** of a mechanical Riemannian manifold is the smooth map

$$K : TM \to \mathbb{R}$$
$$K(v) := \frac{1}{2} g(v, v)$$

[1]

> **Proposition:** Any conservative mechanical manifold $(M, g, -\mathrm{d}U)$ is *positional* and the integral curves on $TM$ preserve the function
>
> $$K + U : TM \to \mathbb{R}$$

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Current note has 1 direct children and 1 total descendants.

- structures based on sets
  - manifolds
    - Manifold with a flow
      - Semi-Riemannian mechanical manifold
        - Potential mechanical manifold $M^N$

And it has 1 siblings.

- structures based on sets
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1. https://people.math.ethz.ch/~acannas/Student_Papers/BSc_Theses/2023_bsc_bachmann_llauger_geometric_mechanics.pdf ↩