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$\mathcal{O}(D)$

Definition. Holomorphic functions on D

We have the space

$$\mathcal{O}(D)$$

of holomorphic functions $D \rightarrow \mathbb{C}$.

Then

- for $p \in [1, \infty]$ let

$$\mathcal{O}^p(D) := \left\{ f \in \mathcal{O}(D) \left| \|f\|_p := \sup_{r \in (0,1)} \left| \frac{1}{2\pi} \int_{rS^1} |f|^p \right|^{\frac{1}{p}} < \infty \right. \right\}$$

is the p -th **Hardy space**

- $\mathcal{O}^\infty(D) := \left\{ f \in \mathcal{O}(D) \left| \|f\|_\infty := \sup_{z \in D} |f(z)| < \infty \right. \right\}$

is the ∞ **Hardy space**

- $\mathcal{O}^N(D) := \left\{ f \in \mathcal{O}(D) \left| \sup_{r \in (0,1)} \frac{1}{2\pi} \int_{rS^1} \log |f| < \infty \right. \right\}$

is the **Nevanlinna class**

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[1]

$$\begin{array}{ccccccc} \mathcal{O}(\overline{D}) & \hookrightarrow & \mathcal{O}(D) \cap \mathcal{C}^\infty(\overline{D}) & \hookrightarrow & \mathcal{O}(D) \cap \mathcal{C}(\overline{D}) & \xhookrightarrow[\| \cdot \|_\infty]{[0, \infty)} & \mathcal{O}^\infty(D) \hookrightarrow \mathcal{O}(D) \\ & & \downarrow & & \downarrow & & \\ & & \mathcal{C}^\infty(S^1) & \hookrightarrow & \mathcal{C}(S^1) & & \end{array}$$

Current note has 5 direct children and 5 total descendants.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Holomorphic functions on spaces over \$\mathbb{C}\$ of dimension 1](#)
 - $\mathcal{O}(D)$
 - [Boundary values of \$\mathcal{O}\(D\)\$](#)
 - $\mathcal{O}(\overline{D})$
 - $\mathcal{O}(D) \cap \mathcal{C}(\overline{D})$
 - $\mathcal{O} \cap L^2(D)$
 - [Reconstructing \$\mathcal{O}\(D\)\$ from its boundary values](#)

And it has 19 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Holomorphic functions on spaces over \$\mathbb{C}\$ of dimension 1](#)
 - [Approximation of holomorphic functions on \$\mathbb{C}\$ by rational functions](#)
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 - [Extending holomorphic functions by reflections](#)
 - [Rotational symmetrization of holomorphic functions](#)
 - [Sheaf of holomorphic functions on \$\mathbb{C}\$](#)
 - $\mathcal{O}(U)$
 - $\mathcal{O}(\mathbb{C})$
 - $\mathcal{O}(D)$
 - $\mathcal{O}^p(H^2_{\mathbb{U}})$
 - $\mathcal{O} \cap L^p$
 - $\mathcal{O}(S^1)$
 - [Zeros and singularities of holomorphic functions](#)

1. [John B. Conway - Functions of One Complex Variable II, 20 Hardy Spaces on the Disk](#) ↔