ⓘ Info

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Naturally reductive homogeneous spaces

Definition. Definition

A homogeneous space ${}^{G}/_{H}$ is **naturally reductive** if

- it is reductive
$$\mathfrak{g} = \mathfrak{h} + \mathfrak{m}$$
- it has a G-invariant Riemannian metric g
- $$\mathrm{g}([X, Z]_{\mathfrak{m}}, Y) = \mathrm{g}(X, [Z, Y]_{\mathfrak{m}})$$

> **Proposition:** If $\left({}^{G}/_{H}, \mathrm{g}\right)$ is naturally reductive then the Levi-Civita connection of g is given by
> $$(\nabla_{X^*} Y^*)_o = -\frac{1}{2}[X, Y]_{\mathfrak{m}}$$

> **Proposition:** If $\left({}^{G}/_{H}, \mathrm{g}\right)$ is naturally reductive then the geodesic passing through o is given by
> $$t \mapsto \exp(tX)(o)$$
> for $X \in \mathfrak{m}$.

[1]

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - Homogeneous manifold
      - Naturally reductive homogeneous spaces

And it has 8 siblings.

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1. cis610-18-sl18.pdf ↩