> **Info**
>
> This note found here
> as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
> created on February 10, 2026 1:37:38 PM,
> and was last modified on February 10, 2026 2:13:10 PM.

Convergent power series

> **Definition. Definition**
>
> Let
>
> $$\sum_{n \in \mathbb{N}} a_n z^n$$
>
> be a formal power series with coefficients in $\mathbb{C}$. We define
>
> $$R(\underline{a}) := \begin{cases} +\infty & \text{when } |a_n|^{\frac{1}{n}} \xrightarrow{n \to \infty} 0 \\ \dfrac{1}{\limsup_{n \to \infty} |a_n|^{\frac{1}{n}}} & \\ 0 & \end{cases}$$

- Let $\underline{a}$ be a sequence.
  - For $z_0 \in \mathbb{C} \setminus \{0\}$, let the sequence $\left\{a_n z_0^n\right\}_{n \in \mathbb{N}}$ be bounded. Then we have

$$\forall n \in \mathbb{N}, |a_n z_0^n| \leq K$$

$$\forall n \in \mathbb{N}, |a_n|^{\frac{1}{n}} \leq \frac{K^{\frac{1}{n}}}{|z_0|}$$

    - So

$$\begin{aligned} \limsup_{n \to \infty} |a_n|^{\frac{1}{n}} &= \lim_{n \to \infty} \underbrace{\sup \left\{ |a_n|^{\frac{1}{n}} \middle| n \geq m \right\}}_{\leq \frac{K^{\frac{1}{n}}}{|z_0|}} \\ &\leq \lim_{n \to \infty} \frac{K^{\frac{1}{n}}}{|z_0|} \\ &= \frac{1}{|z_0|} \end{aligned}$$

- Hence,

$$0 < |z_0| \leq R(\underline{a})$$

- Let $R(\underline{a}) > 0 \iff \exists r > 0$ with $R(\underline{a}) > r$.
  - Then

$$\inf_{m \in \mathbb{N}} \sup \left\{ |a_n|^{\frac{1}{n}} \middle| n \geq m \right\} < \frac{1}{r}$$

meaning there is an m such that

$$\sup \left\{ |a_n|^{\frac{1}{n}} \middle| n \geq m \right\} < \frac{1}{r}$$
$$\forall n \geq m, |a_n|^{\frac{1}{n}} < \frac{1}{r}$$
$$\{a_n r^n\}_{n \in \mathbb{N}} \text{ is bounded}$$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - Hol Equivalent descriptions of holomorphic functions
        - power series Convergent power series

And it has 4 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - Hol Equivalent descriptions of holomorphic functions
        - Cauchy Cauchy integral formula for holomorphic functions
        - d Derivative of holomorphic functions
        - integral Integral of holomorphic differential forms
        - power series Convergent power series