> ⓘ **Info**
>
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> is written (completely with human hands) by Rupadarshi Ray,
> created on October 16, 2024 1:45:06 AM,
> and was last modified on June 12, 2026 8:03:41 PM.

Holomorphic functions meromorphic at infinity

> **Definition. Holomorphic functions meromorphic at infinity**
>
> A holomorphic function
>
> $$f : \{z \in \mathbb{C} \mid |z| > R\} \to \mathbb{C}$$
>
> is **meromorphic at ∞** for all $M > 0$ there is a $r \in (R, \infty)$ such that
>
> $$|z| > r \implies |f(z)| > M$$

- Equivalently

$$\begin{aligned} D^*_{\frac{1}{R}} &\to \mathbb{C} \\ w &\mapsto f\left(\frac{1}{w}\right) \end{aligned}$$

  is meromorphic at $w = 0$

> **Definition. Meromorphic function on the complex plane**
>
> A holomorphic function
>
> $$f : z_0 + D^*_R \to \mathbb{C}$$
>
> is **meromorphic at z_0** if
>
> $$|f(z)| \xrightarrow{z \to z_0} \infty$$
>
> which means
>
> $$\forall M > 0 \exists r \in (0, R) : |z - z_0| < r \implies |f(z)| > M$$

- By
  - Thus for any function f that is meromorphic at z_0, there is a holomorphic $h : z_0 + D_r \to \mathbb{C}$ and positive integer k called the **order of the pole at z_0** such

> that
>
> $$f(z) = \frac{h(z)}{(z - z_0)^k} \text{ on } z_0 + D_r^*$$
>
> Moreover $h(z_0) \neq 0$.

we have for all $w \in D_r^*$

$$f\left(\frac{1}{w}\right) = \frac{h(w)}{w^k} = f(z) = z^n h\left(\frac{1}{z}\right)$$

where h is holomorphic on some D_r, $h(0) \neq 0$ and k is a positive integer called the **order of the pole of f at infinity**.

- Non constant entire functions are meromorphic at infinity.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - Mer at infinity Holomorphic functions meromorphic at infinity

And it has 22 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - approx rat Approximation of holomorphic functions on $\mathbb{C}$ by rational functions
      - embedding into C3 Embedding unit disk D into $\mathbb{C}^3$
      - factorization Factorization of holomorphic functions on $\mathbb{C}$
      - global Global holomorphic functions
      - Hol Equivalent descriptions of holomorphic functions
      - Mer Meromorphic functions on the complex plane
      - Mer at infinity Holomorphic functions meromorphic at infinity
      - modular Modular forms
      - recons mer from stalk Reconstructing meromorphic functions from stalks
      - reflection Extending holomorphic functions by reflections
      - rotation symmetrizer Rotational symmetrization of holomorphic functions
      - sheaf Sheaf of holomorphic functions on $\mathbb{C}$
      - space $\mathscr{O}(U)$

- space C $\mathscr{O}(\mathbb{C})$
- space D $\mathscr{O}(D)$
- space D closed $\mathscr{O}(\overline{D})$
- space D cnt bd $\mathscr{O}(D) \cap \mathcal{C}(\overline{D})$
- space D L2 $\mathscr{O} \cap L^2(D)$
- space H $\mathscr{O}^p(H^2_{\mathrm{U}})$
- space Lp $\mathscr{O} \cap L^p$
- space S1 $\mathscr{O}(S^1)$
- zeros and singularities Zeros and singularities of holomorphic functions