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Branch points and ramification index of holomorphic functions

Definition. **Branch points and ramification index of holomorphic functions**

Let $\Omega \subseteq \mathbb{C}$ be a connected open set and

$$f : \Omega \to \mathbb{C}$$

be a non-constant holomorphic function. Then zeros of f', that is, $(f')^{-1}(0)$ are the set of *critical points* of f and in particular this set has no limit points in Ω as f is holomorphic.

The **ramification index** of f is equivalently

- Around a small disk $B(z_0, r)$ of such a zero point $z_0 \in (f')^{-1}(0)$ which does not contain any other zeros, we have the winding number of the curve $f(\gamma)$
- order of the zero z_0 of f'

If the ramification index at z_0 is greater than 1 then z_0 is called a **ramification point** of f, the *critical value* $f(z_0)$ is called an **(algebraic) branch point** of f.

Equivalently, z_0 is a ramification point of f if

$$f(z) = f(z_0) + (z - z_0)^k g(z),\ k > 1$$

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- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - mapping

        - branch points Branch points and ramification index of holomorphic functions

And it has 9 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - mapping
        - bd derivative Bounding derivative of holomorphic mappings
        - branch points Branch points and ramification index of holomorphic functions
        - contains disks Disks in holomorphic images
        - fibers Fibers of holomorphic mappings
        - locally poly Non-constant holomorphic functions locally look like w^k and are open mappings
        - onto disk Biholomorphic mapping onto disk
        - pole Holomorphic functions on punchered disk
        - singularities are not isolated Holomorphic function whose singularities are not isolated
        - z2 at all rationals A holomorphic map branched over $\mathbb{Q}$