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is written (completely with human hands) by Rupadarshi Ray,
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Fibers of holomorphic mappings

zeros of holomorphic functions are isolated

- If $f : U \to \mathbb{C}$ is a non-constant holomorphic function with

$$f(z) \equiv \sum_{n \geq k} a_n (z - z_0)^n \qquad \text{on } B(r, z_0)$$

for $z_0 \in U$ where k is the smallest positive integer such that $a_k \neq 0$.

  - $\implies$

$$f(z) = (z - z_0)^k g(z) \qquad \text{on } B(r_1, z_0) \subseteq B(r, z_0)$$

  where g is a holomorphic function such that

$$g(z) :\equiv a_k + a_{k+1}(z - z_0) + \dots$$

  - As $g(z_0) = a_k \neq 0$, this means there is a smaller disk on which g is non-zero

$$g \neq 0 \qquad \text{on } B(r_2, z_0)$$

  - $\implies$ $f \neq 0$ on that same punchered disk around z_0
  - Thus zeroes of non-constant holomorphic functions on an open subset of $\mathbb{C}$ are isolated in $\mathbb{C}$, that is, the set of zeros $Z(f)$ is discrete in $\mathbb{C}$.

fibers of holomorphic functions are isolated

(Identity theorem for holomorphic functions on subsets of $\mathbb{C}$) Let

$$f : \Omega(\text{open}) \subseteq \mathbb{C} \to \mathbb{C}$$

be holomorphic. And L be the set of limit points of $f^{-1}(0)$. Then L is both open and closed in Ω.
Therefore if Ω is connected, either $L = \Omega \iff f \equiv 0 \text{ on } \Omega$ or $f^{-1}(0)$ has no limit point in Ω, that is if $f^{-1}(0)$ has a limit point in Ω then $f \equiv 0$.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - mapping
        - fibers Fibers of holomorphic mappings

And it has 9 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - mapping
        - bd derivative Bounding derivative of holomorphic mappings
        - branch points Branch points and ramification index of holomorphic functions
        - contains disks Disks in holomorphic images
        - fibers Fibers of holomorphic mappings
        - locally poly Non-constant holomorphic functions locally look like w^k and are open mappings
        - onto disk Biholomorphic mapping onto disk
        - pole Holomorphic functions on punchered disk
        - singularities are not isolated Holomorphic function whose singularities are not isolated
        - z2 at all rationals A holomorphic map branched over $\mathbb{Q}$