Info

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is written (completely with human hands) by Rupadarshi Ray,
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Extending holomorphic functions by reflections

Proposition: Let $\Omega \subseteq \mathbb{C}$ be an open set which is symmetric under $\mathbb{C}$-conjugation

$$\Omega^{\dagger} = \Omega$$

and

$$f : \Omega \to \mathbb{C}$$

be continuous and holomorphic on $\Omega \setminus \mathbb{R}$. Then f is holomorphic on Ω.

- By

  (Morera's theorem) Any continuous function $f : U \to \mathbb{C}$ on open set $U \subseteq \mathbb{C}$ that satisfies

  $$\int_{\gamma} f(z) \mathrm{d}z = 0$$

  for every $\mathcal{C}^1$ closed curve γ in U must be holomorphic on U.

  for triangles.

(Schwarz reflection principle) Let $\Omega \subseteq \mathbb{C}$ be an open set which is symmetric under conjugation

$$\Omega^{\dagger} = \Omega$$

and

$$f : \Omega \cap H \to \mathbb{C}$$

be a holomorphic function that extends continuously to

$$\Omega \cap \mathbb{R} \to \mathbb{R}$$

Then f extends onto Ω by

$$f(z) = \overline{f(\bar{z})} \text{ for } z \in \bar{\Omega}$$

(uniquely by identity).

$$f(\bar{z}) = \sum_{n \geq 0} a_n (\bar{z} - \bar{z}_0)^n$$

then

$$\overline{f(\bar{z})} = \sum_{n \geq 0} \bar{a}_n (z - z_0)^n$$

is holomorphic on Ω^-

- But for $x \in \Omega \cap \mathbb{R}$ we have $\overline{f(x)} = f(x)$ so f extends continuously on Ω.
- Now by

  > **Proposition:** Let $\Omega \subseteq \mathbb{C}$ be an open set which is symmetric under $\mathbb{C}$-conjugation
  >
  > $$\Omega^\dagger = \Omega$$
  >
  > and
  >
  > $$f : \Omega \to \mathbb{C}$$
  >
  > be continuous and holomorphic on $\Omega \setminus \mathbb{R}$. Then f is holomorphic on Ω.

  it is holomorphic.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - reflection Extending holomorphic functions by reflections

And it has 22 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1

- approx rat Approximation of holomorphic functions on $\mathbb{C}$ by rational functions
- embedding into C3 Embedding unit disk D into $\mathbb{C}^3$
- factorization Factorization of holomorphic functions on $\mathbb{C}$
- global Global holomorphic functions
- Hol Equivalent descriptions of holomorphic functions
- Mer Meromorphic functions on the complex plane
- Mer at infinity Holomorphic functions meromorphic at infinity
- modular Modular forms
- recons mer from stalk Reconstructing meromorphic functions from stalks
- reflection Extending holomorphic functions by reflections
- rotation symmetrizer Rotational symmetrization of holomorphic functions
- sheaf Sheaf of holomorphic functions on $\mathbb{C}$
- space $\mathscr{O}(U)$
- space C $\mathscr{O}(\mathbb{C})$
- space D $\mathscr{O}(D)$
- space D closed $\mathscr{O}(\overline{D})$
- space D cnt bd $\mathscr{O}(D) \cap \mathcal{C}(\overline{D})$
- space D L2 $\mathscr{O} \cap L^2(D)$
- space H $\mathscr{O}^p(H^2_{\mathrm{U}})$
- space Lp $\mathscr{O} \cap L^p$
- space S1 $\mathscr{O}(S^1)$
- zeros and singularities Zeros and singularities of holomorphic functions