ⓘ **Info**

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Sequences of holomorphic functions

uniform limit of holomorphic functions are holomorphic

We shall prove

Let $f_n : U \to \mathbb{C}$ be holomorphic on open $U \subseteq \mathbb{C}$ and

$$f_n \xrightarrow{\text{uniformly as } n\to\infty} f \text{ on } K$$

***for every $K \subset U$ compact* then f is holomorphic in U.**

Choose a ball of some radius r around $z_0 \in U$

$$f_n(z) = \frac{1}{2\pi i}\int_{S^1_{r,z_0}} \frac{f_n(w)}{w-z}\mathrm{d}z \xrightarrow{\text{uniformly}} \frac{1}{2\pi i}\int_{S^1_{r,z_0}} \frac{f(w)}{w-z}\mathrm{d}z$$

on $|z-w| \leq \frac{r}{2}$.

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holomorphic $\iff$ uniform limit of rationals

Approximation of holomorphic functions on $\mathbb{C}$ by rational functions

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Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - space $\mathscr{O}(U)$
        - seq Sequences of holomorphic functions

And it has 2 siblings.

- stamp stamp
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    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - space $\mathscr{O}(U)$
        - pre-cpt Pre-compact subsets of $\mathscr{O}(U)$
        - seq Sequences of holomorphic functions