Info

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$$\mathscr{O}(D) \cap \mathcal{C}(\overline{D}) \xrightarrow{\text{res}} \mathcal{C}(S^1)$$

Definition. **The disk algebra is**

$$\mathscr{O}(D) \cap \mathcal{C}(\overline{D}) := \left\{ f \in \mathcal{C}(\overline{D}) \middle| f|_D \in \mathscr{O}(D) \right\}$$

$$\begin{aligned} \text{res} : \mathscr{O}(D) \cap \mathcal{C}(\overline{D}) &\to \mathcal{C}(S^1) \\ \sum_{n\in\mathbb{N}} a_n z^n &\mapsto \sum_{n\in\mathbb{N}} a_n e^{in\theta} \end{aligned}$$

is an isometric isomorphism onto the closed subalgebra

$$\left\{ g \in \mathcal{C}(D) \middle| \forall n < 0, \hat{g}(n) = 0 \right\} \subseteq (\mathcal{C}(S^1), \|\ \|_\infty)$$

- $$\overline{\mathbb{C}[z] \cap (\mathcal{C}(\overline{D}), \|\ \|_\infty)} = \mathscr{O}(D) \cap \mathcal{C}(\overline{D})$$
- By maximum modulus, for $f \in \mathscr{O}(D) \cap \mathcal{C}(\overline{D})$ there is a $z_0 \in S^1$ such that

  $$\|f\|_\infty = f(z_0) = \|f|_{S^1}\|_\infty$$

  so res is an isometric embedding.

Proposition:

$$\begin{aligned} \sigma_{\mathscr{O}(D)\cap\mathcal{C}(\overline{D})}(f) &= f(D) \\ \sigma_{\mathcal{C}(S^1)}(f) &= f(S^1) \end{aligned}$$

- $\operatorname{maxSpec}\mathscr{O}(D) \cap \mathcal{C}(\overline{D}) \cong ?\overline{D}$
- $\mathscr{O}(D) \cap \mathcal{C}(\overline{D}) \subseteq \mathscr{O}^\infty(D)$

$C : \mathcal{C}(S^1) \to \mathscr{O}(D) \cap \mathcal{C}(\overline{D})$

$G : \mathcal{C}(S^1) \to \mathscr{O}(D) \cap \mathcal{C}(\overline{D})$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - space D cnt bd $\mathscr{O}(D) \cap \mathcal{C}(\overline{D})$

And it has 22 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - approx rat Approximation of holomorphic functions on $\mathbb{C}$ by rational functions
      - embedding into C3 Embedding unit disk D into $\mathbb{C}^3$
      - factorization Factorization of holomorphic functions on $\mathbb{C}$
      - global Global holomorphic functions
      - Hol Equivalent descriptions of holomorphic functions
      - Mer Meromorphic functions on the complex plane
      - Mer at infinity Holomorphic functions meromorphic at infinity
      - modular Modular forms
      - recons mer from stalk Reconstructing meromorphic functions from stalks
      - reflection Extending holomorphic functions by reflections
      - rotation symmetrizer Rotational symmetrization of holomorphic functions
      - sheaf Sheaf of holomorphic functions on $\mathbb{C}$
      - space $\mathscr{O}(U)$
      - space C $\mathscr{O}(\mathbb{C})$
      - space D $\mathscr{O}(D)$

- space D closed $\mathscr{O}(\overline{D})$
- space D cnt bd $\mathscr{O}(D) \cap \mathcal{C}(\overline{D})$
- space D L2 $\mathscr{O} \cap L^2(D)$
- space H $\mathscr{O}^p(H^2_{\mathrm{U}})$
- space Lp $\mathscr{O} \cap L^p$
- space S1 $\mathscr{O}(S^1)$
- zeros and singularities Zeros and singularities of holomorphic functions