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> is written (completely with human hands) by Rupadarshi Ray,
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Analytic functions on the circle $\mathcal{C}^\omega(S^1)$

Consider the space of analytic functions on the circle

$$\mathcal{C}^\omega(S^1) \hookrightarrow \mathcal{C}^\infty(S^1)$$

$\operatorname{maxSpec} \mathcal{C}^\omega(S^1)$ and classification of ideals

- Let $I \subseteq \mathcal{C}^\omega(S^1)$ be an ideal not contained in any $\mathfrak{m}_p, p \in S^1$.
  - Thus for every $p \in S^1$ there is a $f(p) \neq 0$ implying f is non-zero in a onb of p, so

    $$\left\{S^1 \setminus f^{-1}(0) \middle| f \in I \setminus \mathfrak{m}_p, p \in S^1\right\}$$

    is an open cover of S^1.
  - Since S^1 is compact, there is a finite subcover, so there are finitely many $f_1, \ldots, f_n \in I$ such that $\left\{S^1 \setminus f_i^{-1}(0)\right\}$ cover S^1.
  - Thus

    $$f_1\overline{f}_1 + \cdots + f_n\overline{f}_n \in I$$

    has no zeros, so it is invertible.

Hence,

> **Proposition:**
>
> $$\operatorname{maxSpec} \mathcal{C}^\omega(S^1) \cong S^1$$

[1]

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- space space
    - R The real and complex numbers
        - sphere Spheres
            - 1 S^1
- stamp stamp
    - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
        - An
            - S1 Analytic functions on the circle

And it has 1 siblings.

- stamp stamp
    - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
        - An
            - S1 Analytic functions on the circle

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